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AR Number: AR-001-029

LEVEL

AD A082086

DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
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EDGECLIFF, N.S.W.

DTIC
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RANRL REPORT 6/78

SHIP MOTION MEASUREMENTS MADE ON AN ATTACK CLASS PATROL BOAT (HMAS BOMBARD)

BY

J. L. THOMPSON

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COPY No. 49

NOVEMBER 1979

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Page 1

(11)

DEPARTMENT OF DEFENCE

RAN RESEARCH LABORATORY

(9) Final rept. Oct 76 – Dec 78

(14) RANRL-~~Report~~-6/78

(6)

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(10) J. L. Thompson

(11) Nov

(12) 127

S U M M A R Y

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Approved for public release.

POSTAL ADDRESS: The Superintendent, RAN Research Laboratory
P.O. Box 706, Darlinghurst N.S.W. 2010

406 608

1B

DOCUMENT CONTROL DATA SHEET

Security classification of this page: UNCLASSIFIED

1. DOCUMENT NUMBERS: a. AR Number: AR 001-029 b. Document Series and Number: RANRL Report c. Report Number: 6/78	2. SECURITY CLASSIFICATION: a. Complete document: UNCLASSIFIED b. Title in isolation: UNCLASSIFIED c. Summary in isolation: UNCLASSIFIED

3. TITLE: Ship Motion Measurements made on an Attack Class Patrol Boat (HMAS BOMBARD)

4. PERSONAL AUTHOR(S): J. L. Thompson	5. DOCUMENT DATE: DEC 78 6. TYPE OF REPORT AND PERIOD COVERED: Final OCT 76 - DEC 78
7. CORPORATE AUTHOR(S) RAN Research Laboratory P.O. Box 706 DARLINGHURST NSW 2010	8. REFERENCE NUMBERS: a. Task: C7/76 (A3) b. Sponsoring Agency: NAVY c. Approved: W. J. Hunter SSRL 9. COST CODE:
10. IMPRINT (Publishing establishment) RAN Research Laboratory	11. COMPUTER PROGRAM(S): (Title(s) and Language(s))

12. RELEASE LIMITATIONS (of the document): in addition to the distributionlist

AUSTRALIA: Approved for public release.

OVERSEAS: N.O. [ ] P.R. [1] A [ ] B [ ] C [ ] D [ ] E [ ]

13. ANNOUNCEMENT LIMITATIONS (of the information on this page):

No limitation

14. DESCRIPTORS: Pitching Ocean Waves Roll Patrol Ship	15. COSATI CODES: 2011

16. SUMMARY: This report describes ship motion trials that were carried out on the Attack Class Patrol Boat HMAS BOMBARD off Sydney during October 1976. The patrol boat carried out runs at 10, 15 and 19 knots into head, beam and following seas. Data on roll, pitch, lateral and vertical acceleration are presented. Waveheight measurements were made with a Datawell Waverider Buoy. This report presents the spectra of roll, pitch, vertical and lateral acceleration and waveheight. An analytical method of obtaining the spectrum of the rate of change of a function given its spectrum is derived. This resulting transformation is used to obtain the spectrum of the rate of change of vertical acceleration which is an important quantity in seakeeping and motion sickness studies. The transfer function of the patrol boat when it is considered as a system with the waveheight spectrum as the input is obtained.

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"AS A QUARTERMASTER IN CORVETTES ONCE SAID DURING THE WAR:

'THESE SHIPS IS ALL RIGHT. THE ONLY TROUBLE WITH THESE SHIPS IS THAT THE BLOKE WHAT INVENTED 'EM, DIDN'T INVENT THE BLOKES THAT SHOULD GO WITH 'EM'."

SHIPBUILDING AND SHIPPING RECORD

28 NOV 1957. p.695

1. INTRODUCTION

This report describes a Trial that was carried out off Sydney during October 1976 to measure the motion of an Attack Class Patrol Boat HMAS BOMBARD. The Patrol Boat was instrumented to measure roll, pitch and vertical and lateral accelerations at several points and a Datawell Waverider Buoy was used to measure waveheight.

These measurements were carried out for the Director of Naval Ship Design to obtain data to assist in the validation of the SCORES digital computer program. This program calculates the vertical and lateral plane motions of a ship in waves. The accuracy of ship motion theory for frigate size vessels and above has been well demonstrated but the relationship between theory, model and fullscale motions for small vessels is almost unexplored in the ship design literature.

During the trial runs were carried out at various engine r.p.m. and at different headings to the prevailing swell. Roll, pitch and accelerations were recorded on magnetic tape for later analysis. A similar series of measurements was carried out on the Attack Class Patrol Boat HMAS AWARE in June 1975 (Porter and others, 1976). In this trial, ship motion data were recorded on a paper chart and the subsequent analysis proved to be very time consuming and spectral analysis was not practical.

In this report data are presented on the waveheight spectra measured during the trial together with data on the spectra of roll, pitch, and vertical and lateral acceleration. Data on the rate of change of vertical acceleration are also presented. This parameter is considered to be important in motion sickness studies.

2. INSTRUMENTATION

2.1 ABOARD PATROL BOAT

The instrumentation aboard HMAS BOMBARD to measure ship motion was similar to that which was used in the previous trial (Porter and others, 1976) so only a brief description is given here. Pitch and roll were measured with a Giannimi Type 3111 vertical gyro and accelerations were measured with Giannimi Type 24128 accelerometers. The positions of these transducers are shown in figure 1 and the parameters measured at each location were:

Wheelhouse	-	lateral acceleration
For'd	-	vertical and lateral acceleration

CO's cabin	-	vertical and lateral acceleration
		pitch
		roll

The accelerometer measuring lateral acceleration in the for'd position became unserviceable during the trial and no data were obtained from it. The sensitivities of the tranducers are:

Acceleration	1.4	volts/g
Roll	0.037	volts/degree
Pitch	0.1	volts/degree

The transducer outputs were recorded on magnetic tape using an AMPEX AR 200, 7 track tape recorder in the FM mode. The locations of the transducers were identical to those in the previous trial and the details of these positions are given in Porter and others (1976).

The patrol boat instrumentation was supplied, installed and operated during the trial by personnel from the Aeronautical Research Laboratory, Melbourne.

2.2 WAVEHEIGHT MEASUREMENTS

Waveheight was measured with a Datawell Waverider Buoy which was moored in 125 metres of water in a position 10 nautical miles east of the entrance to Sydney Harbour as shown in figure 2.

The Waverider buoy is spherical with a diameter of 0.7 m and measures waveheight by following the movement of the water surface. The output of the accelerometer is integrated twice to give the vertical displacement of the buoy. As the buoy and its mooring are designed to follow the waves the vertical displacement is equal to the waveheight. The details of the mooring used with the buoy are shown in figure 3.

The buoy was deployed from HMAS BOMBARD on 25 October 1976 and recovered using a Torpedo Recovery Vessel (TRV) on 2 November 1976. The response of the buoy to waves is shown in figure 4. The transfer function is unity between about 0.06 Hz and 0.5 Hz. Below 0.06 Hz the fall off is caused by the lack of response of the integration circuits at low frequencies. The peak at 0.8 Hz is due to the resonance of the buoy-mooring system. At higher frequencies the dimensions of the buoy became comparable to the wavelength of the waves and the buoy cannot follow them.

The waveheight data from the buoy was transmitted over a radio link to a receiver which was set up at HMAS WATSON for the duration of the trial.

The waveheight was displayed on a chart recorder. At the receiver output a DC voltage which corresponded to waveheight was available. The system sensitivity was 1 volt/metre. The power density spectrum of the waveheight was obtained by analysing this signal in a Spectral Dynamics SD 360 analyser. An FM tape recording was also made of the waveheight.

2.3 NAVIGATION

Accurate measurement of the speed of the patrol boat was essential during this trial. The Mini-Ranger III System was used to fix the position of the patrol boat during runs so that the average speed could be determined.

The Mini-Ranger III System (MRS III) uses a C Band transmitter located on a mobile unit (in this case HMAS BOMBARD) to interrogate two reference station transponders which were situated ashore. The elapsed time between interrogation and the reply from the transponders is used to determine the range to each reference station. The MRS III operates at line-of-sight ranges of up to 20 nautical miles and is capable of an accuracy of better than 3 metres. A printer was used with the mobile unit located on HMAS BOMBARD to give a hardcopy record of the time against the range to each of the transponders.

The two reference station transponders were located (see fig.2) on Barranjoey Head Light and at Macquarie Light. Each transponder was clamped onto the railing of a balcony just below the light. The transponder at Barranjoey was 113 metres above sea level and the one at Macquarie was 105 metres. If the antenna on HMAS BOMBARD is assumed to be 6 metres above sea level then it can be shown that the maximum free-space line-of-sight transmission is approximately 27 nautical miles for the transponder at Macquarie Light. The range from the Barrenjoey Light will be slightly greater.

The two lighthouses were seventeen nautical miles apart which was an adequate separation to give accurate fixes over quite a large area. A suitable power supply (240 volts, 50 Hz) was readily available at both lighthouses. The two stations were set up during the week before the measurements commenced and were unattended during the trial.

3. DETAILS OF MEASUREMENTS MADE

3.1 PATROL BOAT MOTION

The general aim of the trial was to measure patrol boat motion at several different speeds for beam, head and following seas. The details of the eleven runs carried out are listed in table 1. This table shows the start and stop time of each run, engine r.p.m., the nominal and measured speed, the course during the run and the type of run. The measured speed and course were obtained from reconstructed plots.

On 26 October 1976 the conditions were judged by the Commanding Officer of BOMBARD to be too rough and only two runs were carried out. On 27 October the seas had abated and nine runs were completed. Plots of the eleven runs are shown in figures 5 and 6. During the extreme southern portions of runs 3 and 4 of 27 Oct 76, the range of BOMBARD from the Barranjoey station was greater than the allowable range gate of 37 km and the track shown as dotted was reconstructed by dead reckoning. The duration of each run was approximately 30 minutes. It will be shown in section 4.1 of this report that a record length of this duration was necessary to obtain a reasonable estimate of the spectra. The nominal speed was obtained from the engine r.p.m. vs speed tables held on board HMAS BOMBARD. These tables are normally obtained during still water trials.

A copy of the log kept by the Trials Officer during the trial is included in this report as Annex E.

3.2 WAVEHEIGHT

The Waverider Buoy was moored at 1115 on 25 October and the first waveheight spectrum was obtained between 1144 and 1210 that day. It will be shown in section 4.1 that a 26 minute record length of waveheight data was necessary to give a reasonable estimate of the waveheight spectra. In figure 7 are shown the times during which waveheight spectra were measured together with the times of the patrol boat runs. All times in this report are KILO.

TABLE 1: PATROL BOAT SEAKEEPING TRIAL - DETAILS OF RUNS

Run No.	Date	Start Time K	Stop Time K	r.p.m.	Nominal Speed knots	Measured Speed knots	Measured Course	Type of Run
1	26 OCT 76	0949	1015	1000	15.0	14.45	099	Head Sea
2	"	1017	1047	1000	15.0	15.0	273	Following
1	27 OCT 76	0947	1017	1250	19.0	17.8	089	Head
2	"	1020	1049	1250	19.0	19.0	279	Following
3	"	1051	1121	1250	19.0	17.0	184	Beam
4	"	1132	1201	600	10.5	10.5	359	Beam
5	"	1204	1235	600	10.5	9.3	088	Head
6	"	1237	1307	600	10.5	10.4	275	Following
7	"	1316	1346	1000	15.0	15.0	004	Beam
8	"	1348	1418	1000	15.0	14.0	107	Head
9	"	1419	1449	1000	15.0	15.0	286	Following

4. DATA ANALYSES

4.1 SPECTRAL ANALYSIS OF WAVEHEIGHT AND PATROL BOAT MOTION RECORDS

The data from the Waverider buoy and from the roll, pitch and acceleration transducers on the patrol boat are random functions. In this trial data were measured over periods of approx. 30 minutes and it has been assumed that the waveheight spectra remained constant during this period. This is a reasonable assumption as the wind which generated the waveheight did not change much over these time intervals. Similarly as the patrol boat was kept on a steady course and speed and as the previously assumed constant waveheight spectra can be considered as a random forcing function then the patrol boat motions can also be considered to be stationary.

These stationary random data were processed to obtain the power spectral density function. The waveheight data were analysed in real time at HMAS WATSON using a Spectral Dynamics SD-360 Digital Signal Processor . The data from the patrol boat were analysed later at RANRL using the same processor.

The Spectral Dynamics SD-360 Digital Signal Processor carries out a 1024 point Fast Fourier Transform on a block of input data. It then averages up to 4096 spectra to give a more reliable estimate.

The data were initially examined to obtain an estimate of the frequencies in which most of the energy was concentrated. It was found that for both the waveheight data and the patrol boat motion data that there was no significant energy above 1 Hz. However the lowest frequency range on the SD-360 is 10 Hz, thus a 1024 point FFT gives 512 frequency lines (ie a frequency resolution in this case of 0.02 Hz). Therefore most of the energy would be concentrated in the lower 10% of the frequency range. In order to avoid this the SD-360 was externally clocked at 10.24 Hz and aliasing was prevented with a Rockland low-pass filter with a cut-off frequency of 4 Hz. At this sampling rate it took 100 seconds to read a block of 1024 points into the memory of the SD-360. The resolution bandwidth B_e is given by $B_e = 1/T_e$ and thus $B_e = 0.01$ Hz.

It is shown in Bendat and Piersol (1971) that the spectral estimates calculated from a single record of a nondeterministic time series of length T are subject to substantial random errors. Increasing the length of the record does not reduce the errors as the distribution function defining the random error as the estimate is independent of the record length. However, the number of spectral components in the estimate is increased. One method of reducing the random error of the estimate is to smooth over an ensemble of estimates. This is done by computing individual estimates from "q" individual sample records and then averaging the "q" estimates at each frequency of a spectral component. If the total record length is T then $T = q\ T_e$ and it can be shown that the sampling distribution of the smoothed estimate is approximately chi-squared with $n = 2\ B_e\ T = 2q$ degrees of freedom. In figure 8 which is taken from Jenkins and Watts (1968) can be seen the way in which the confidence intervals vary as the number of degrees of freedom increase. It can be seen that if the number of degrees of freedom is low then the errors in the spectral estimates can be quite large. This figure also shows that the reduction in error as the degrees of freedom increase beyond about 70 is quite small. For example if n = 4 the multiplying factors for 95% confidence intervals are 0.36 and 8 which ıld be an unacceptable error in most applications. If n is increased to 30 the factors become 0.64 and 1.8 however a further increase to 100 only reduces the factors to 0.77 and 1.35 . A law of diminishing returns clearly applies here as there is very little increase in accuracy at the cost of a considerable increase in analysis effort and the quantity of input data.

One of the limitations of the SD-360 analyser is that the number of spectra averaged must be an integral power of 2 ie $q = 2^N$ where N = 0, 1, 2, ... , 12. For the spectra presented in this report N = 4 ie 16 spectra were averaged and the estimates had 32 degrees of freedom and the values of the multipliers for 95% confidence limits were 0.64 and 1.75 . The total record length of data to obtain each spectrum was 100 seconds x 16 = 26 minutes 40 seconds. The selection of this record length represents a compromise between increasing the accuracy of the spectral estimates by increasing T and a possible decrease in accuracy since the data may become non-stationary as the record length increases due to changes in wind velocity. It is quite clear however that for record lengths less than about 10 minutes (n = 12) that the errors would have been unacceptably large.

The methods used in calibrating the spectra for roll, pitch and acceleration are given in Annex A and that for the waveheight in Annex B.

4.2 ROOT MEAN SQUARE VALUES

If H_{rms} is the root mean square waveheight and G(f) is the waveheight power spectrum then

$$H_{rms}^2 = \int_0^\infty G(f)\, df$$

The integral was numerically evaluated using Simpson's Rule with frequency increments of 0.01 Hz. The r.m.s values of pitch, roll and vertical and lateral acceleration were similarly calculated.

4.3 RATE OF CHANGE OF VERTICAL ACCELERATION

In this trial vertical acceleration was measured at two locations in the patrol boat. These were in the CO's cabin and in the for'd position. An important parameter in sea keeping and motion sickness studies is the rate of change of vertical acceleration. This report therefore presents data on the spectra and the r.m.s. values of the rate of change of vertical acceleration for various speeds and headings.

One method of obtaining the spectrum of the time deritive of a signal is to differentiate it using an operational amplifier with a capacitor as the input impedance and a resistor in the feedback path. The spectrum of this differentiated signal is then obtained using a spectrum analyser. This method however has the undesirable feature that any high frequency noise in the signal is amplified more than the low frequency noise. Problems with amplifier stability can also be encountered.

It is shown in Annex A that the power spectrum of the deritative of a time series can be obtained in a straightforward way from the spectrum of the original time series. In particular if $G_x(f)$ is the power spectrum of a time series x(t) and if $G_{\dot{x}}(f)$ is the spectrum of the derivative of the time series $\dot{x}(t)$ then

$$G_{\dot{x}}(f) = 4\pi^2 f^2 G_x(f) \qquad (1)$$

When the frequency is given by $f = \frac{1}{2\pi}$ ie $4\pi^2 f^2 = 1$ (f = 0.16 Hz)

$$\text{then } G_{\dot{x}}(f) = G_x(f) \qquad (2)$$

In order to further investigate the two methods of obtaining the power spectrum of the derivative using the transformation given by equation 1 the vertical accelerometer signal was differentiated using an operational amplifier. The dual transform mode of the SD-360 was used to simultaneously obtain the spectra of both the original signal and of its derivative. The equipment used is shown in figure 9. The gain of the differentiator was set so that for an input signal at a frequency of 0.16 Hz the spectral levels were the same for the signal (channel A) and its derivative (channel B).

Results for a typical vertical acceleration spectrum are shown in figure 10. Also shown on this figure are the spectra of the rate of change of vertical acceleration obtained by the two methods just discussed. The agreement between the two spectra is good over the frequency range 0.07 Hz to 0.4 Hz. Above 0.4 Hz the transformation method gives slightly higher spectral levels than by using the electronic differentiator. The spectra presented in this report have been obtained using the transformation method. The main reasons for this are doubts about the accuracy of electronic differentiation over such a wide frequency range and the comparative ease and speed with which the spectra can be obtained using the transformation method.

4.4 TRANSFER FUNCTION OF THE PATROL BOAT

If the patrol boat is moving with a fixed heading relative to the prevailing sea and with fixed r.p.m. then it may be considered to be a system with an input and an output. The input is the waveheight which is a random function of time. The outputs are the six motions of the patrol boat (roll, pitch, yaw, heave, sway, surge). If the system is linear and time-invariant and the input is stationary then it can be shown (Laning and Battin 1956) that

$$G_o(f) = |H(f)|^2 G_w(f) \qquad f \geq o \qquad (3)$$

where $G_o(f)$ = Power spectral density of output,
pitch and roll (degree^2/Hz)
acceleration (g^2/Hz)

$G_w(f)$ = Power spectral density of waveheight (m^2/Hz)

$H(f)$ = Transfer function of the patrol boat for a given parameter (eg pitch) at a fixed r.p.m. and aspect.

The units of $|H(f)|^2$ are: pitch and roll (degree /m)^2

acceleration $(g/m)^2$

Since the ordinates of the spectra presented in this report are plotted on a logarithmic scale the function $|H(f)|^2$ can readily be obtained by subtraction.

5. RESULTS

5.1 WAVEHEIGHT

Between 25th and 27th October 1976 ten waveheight records were taken. In figure 7 the times these records were taken are shown. It can be seen that during the period on 27 October when nine patrol boat runs were carried out that almost continual wave records were taken. Records of wind speed and direction during the trials period are presented in figure 11. These records were obtained from the Maritime Services Board Signal Station at South Head. The staff on duty at this Station record the wind speed and direction every hour. The wind velocity data are plotted in vector form in figure 11 and it shows the magnitude of the wind and the direction the wind is blowing from at hourly intervals from 0000 24 OCT to 2359 27 OCT 76. Where no wind speed is given the Signal Station records indicated light airs.

These wind speed records show that on Sunday 24th October the wind increased to 20-30 knots from the north at about 1800 and remained quite high until about 1000 on the 25th. The wind then moderated and during the remainder of that day and during the 26th the winds varied between light airs and 10-15 knots although for a short period on the 26th the wind gusted to 28 knots.

The high seas generated by the winds of the 24th and 25th therefore decreased during the latter part of the 25th and during the 26th and by the 27th the seas were such that the trial could commence. The main series of runs for the trial were carried out on the 27th and during this trials period the wind speed steadily increased and blew from a generally southerly direction, reaching a speed of 28 knots at 1700K.

In figures 12 to 16 are shown the waveheight data as recorded on a chart recorder during the trial. In most cases these chart records cover the same periods used to compute the waveheight spectra. No chart record was obtained for the period commencing 1230 on 27 OCT 76 because of a problem with the ink in the chart recorder.

The ten waveheight spectra are shown in figures 17 to 26. The 95% confidence limits and the R.M.S. waveheight are also shown on each of the spectra. The Pierson-Moskowitz waveheight spectrum which describes a "fully developed" sea has been plotted in Annex D as a function of wind speed. The analytical expression for this spectrum is also given in Annex D. The Pierson-Moskowitz spectra have been plotted on figures 17 to 26 for several values of windspeed.

All the measured waveheight spectra had similar characteristics in that for frequencies above about 0.15 Hz the spectra have a slope of f^{-5}. The slight peak at about 0.7 Hz is caused by resonance in the buoy-mooring system.

It is quite apparent from figure 11 that the only period during which the wind velocity was fairly constant for a sufficient duration for the sea to be "fully developed" was from 1800 on 24 OCT to about 1000 on 25 OCT 76. The average wind speed during this period was about 25 knots and a wind of this speed requires about 15 hours for the sea to reach an equilibrium state. It can be seen from figure 17 that the agreement between the measured waveheight spectrum and the Pierson-Moskowitz spectrum for a wind speed of 25 knots is quite good.

At all other times during the trial the wind velocity tended to be variable and the seas did not have time to "fully develop" but rather were being generated or were decaying.

The spectrum measured at 1144 on 25 OCT has a single peak at a frequency of 0.09 Hz in a period of 11 seconds. At 0932 the next day the spectrum shows two distinct peaks at frequencies of 0.14 Hz and 0.07 Hz. The energy at 0.14 Hz arises from the locally generated sea whereas that at

0.07 Hz is generated by a distant storm and is called swell. An inspection of the corresponding waveheight record in figure 12 clearly shows a component at 4 cycles/minute ie about 0.07 Hz. Spectra taken at 1040 and 1614 again show the components at 0.07 Hz and 0.14 Hz however the swell component shows a steady decrease in energy throughout the day. The spectrum measured at 1001 on 27 OCT shows no evidence of the swell component.

During 27 OCT six spectra were measured between 1001 and 1451. The wind steadily increased during this period from 10 knots at 0900 to 25 knots at 1400 and its direction varied between SSE and SE. Thus the criterion for an equilibrium sea were not satisfied. The energy of the spectral peak steadily increased, however the frequency of this peak remained constant at 0.11 Hz. The r.m.s. waveheight increased from 0.61 m at 1001 to 0.95 m at 1451.

In figure 27 is plotted the wind speed and the root-mean-square waveheights that were measured during the trial. This shows clearly the increasing wind speed during the 27th and the corresponding increase in r.m.s. waveheight.

5.2 PATROL BOAT SPEED

The measured speed of the patrol boat for each of the eleven runs is shown in table 2 and these data are shown plotted in figure 28. This figure shows that in general for a given r.p.m. the speed for a head sea is lower than for either a beam or a following sea. The current in the trials area is generally in a direction parallel to the coast and has a magnitude of a few knots. The interpretation of the speed during the beam sea runs (ie runs 3, 4, and 7 on the 27 OCT) is made difficult as the magnitude of the current was not measured.

Figure 29 shows the ratio of the measured speed to the nominal speed for the three different headings as the engine r.p.m varies. In the case of the following sea the measured speed is equal to the nominal speed. For the head sea however it can be seen that as the engine r.p.m are increased the ratio of measured to nominal speed increases.

TABLE 2 : SUMMARY OF RESULTS OF PATROL BOAT MOTION

Run No.	Date/ Time K	Type	r.p.m.	Nominal Speed (kts)	H_{rms} (m)	Vertical Acceleration (g) For'd	Vertical Acceleration (g) CO Cabin	Lateral Acceleration (g) CO Cabin	Lateral Acceleration (g) W'house	Roll (degrees) r.m.s.	Pitch (degrees) r.m.s.	Rate of change of vertical accel. (For'd) (g/sec)
1	26 OCT 0949	Head	1000	15	0.96	0.31	0.18	0.059	0.072	3.0	5.6	0.65
2	26 OCT 1017	Foll.	1000	15	0.90	0.067	0.049	0.083	0.12	4.7	3.5	0.15
1	27 OCT 0947	Head	1250	19	0.61	0.25	0.14	0.059	0.077	3.4	3.7	0.52
2	27 OCT 1020	Foll.	1250	19	0.62	0.069	0.071	0.056	0.061	3.9	2.0	0.17
3	27 OCT 1051	Beam	1250	19	0.63	0.11	0.085	0.072	0.091	4.8	2.2	0.24
4	27 OCT 1132	Beam	600	10	0.65	0.085	0.045	0.062	0.12	4.2	2.0	0.14
5	27 OCT 1204	Head	600	10	0.67	0.16	0.088	0.051	0.089	2.5	4.1	0.30
6	27 OCT 1237	Foll.	600	10	0.69	0.090	0.057	0.039	0.051	2.6	2.7	0.17
7	27 OCT 1316	Beam	1000	15	0.67	0.079	0.044	0.076	0.104	4.6	2.1	0.16
8	27 OCT 1348	Head	1000	15	0.85	0.20	0.12	0.049	0.071	2.7	3.5	0.44
9	27 OCT 1419	Foll.	1000	15	0.93	0.060	0.049	0.069	0.082	3.7	3.0	0.14

5.3 PATROL BOAT MOTION

Chart records showing roll, pitch, vertical and lateral acceleration for each of the eleven runs are shown in figures 30-40. Each figure shows data for about 150 seconds which is only 10% of the total record length of 26 minutes that was used to compute the spectra. Some care should therefore be taken in drawing quantitative conclusions about patrol boat motions based on these figures. The speed shown on each figure is nominal speed corresponding to the engine r.p.m. used during the run.

A summary of the results of the patrol boat motion is shown in table 2. The details of each run and the r.m.s. waveheight corresponding to the mid-point of each run are tabulated. This r.m.s. value was obtained by interpolating between the actual r.m.s. waveheight measurements. Also listed are the r.m.s. values of roll, pitch, vertical and lateral acceleration and rate of change of vertical acceleration. The lateral acceleration of a point on the patrol boat is mainly a combination of the roll, sway, and yaw components of motion. Similarly vertical acceleration mainly results from heave, and pitch and roll.

5.3.1 Roll. The roll spectra are shown plotted in figures 41 to 44. In the runs carried out at 10 kts and 15 kts the spectra have a peak at a frequency of 0.17 Hz (ie a roll period of 6 seconds). The 19 knot run has a fairly flat spectrum up to a frequency of about 0.17 Hz. Above this frequency all the roll spectra show the energy falling off with a slope of f^{-5}. This is the same slope as that of the measured wave-height spectra for frequencies above about 0.1 Hz.

At each of these speeds the roll energy for the beam sea runs is greater than that for the head or following sea runs. In general the roll for the head sea runs is greater than that for the following sea runs.

The roll per unit waveheight has been plotted in figure 45. This parameter is obtained by dividing the r.m.s. roll by the r.m.s. waveheight. In the case of a head sea the roll shows a minimum at 1000 r.p.m. whilst for following and beam seas roll increased as the r.p.m. increased.

5.3.2 Pitch. The pitch spectra are shown plotted in figures 46-49. A common feature is that the energy is concentrated in two peaks, one at about 0.25 Hz and another at a lower frequency varying between 0.04 and 0.07 Hz.

The peak at about 0.25 Hz is due to the natural pitch frequency of the patrol boat. The amplitude of the pitch at this frequency is greatest for the head sea runs.

The lower frequency peak is due to the rate at which the patrol boat encounters the sea and it would be expected that this encounter frequency would be a function of patrol boat speed and heading to the prevailing sea.

For gravity waves the following relationships apply

$$C^2 = \frac{gL}{2\pi}$$

$$L = \frac{gT^2}{2\pi}$$

$$L = CT$$

where C = Phase speed of the wave (celerity) (m/sec)

L = wavelength (m)

T = period (sec)

A typical wave of frequency 0.1 Hz therefore has a wavelength of L = 156 m and a celerity C = 15.6 m/sec, ie 30.3 knots. Therefore for a following sea the wave peaks travel faster than the patrol boat ie an overtaking sea. If the speed of the patrol boat is V knots (ie 0.51 V m/sec) then if F_E is the frequency at which successive wave peaks are encountered in a following sea then F_E is given by:

$$F_E = \frac{C - 0.51V}{L}$$

$$= \frac{\frac{gT}{2\pi} - 0.51V}{\frac{gT^2}{2\pi}}$$

$$= f\left(1 - \frac{fV}{3.06}\right) \qquad \text{where } f = \frac{1}{T} = \text{frequency of wave (Hz)}$$

This relationship is plotted in figure 50 for waves of several different frequencies. If the sea has a frequency of 0.1 Hz and the speed of the patrol boat is 5 knots then the encounter frequency is 0.083 Hz (a period of 12 seconds). If the speed increases to 20 kts then the encounter frequency falls to 0.035 Hz (ie 29 seconds period). Of course if the patrol boat speed is equal to the wave celerity then the boat will remain fixed relative to the wave and the encounter frequency is zero. This situation can only occur in the ideal sea (ie unidirectional and monochromatic).

The measured encounter frequencies for following seas are shown in figure 51. During the runs at 10, 15 and 19 knots the dominant wave frequency was 0.11 Hz. The agreement between these three points and the theoretical curve is very good. During one of the 15 knot runs the spectrum of the waveheight showed two peaks, at frequencies of 0.070 and 0.155 Hz. A small section of the theoretical curves corresponding to these wave frequencies has been plotted. Again the agreement between the measured and calculated frequencies is very good.

Similarly it can be shown that for a head sea

$$f_E = f\left(1 + \frac{f\,V}{3.06}\right)$$

This expression has been plotted in figure 50 for a wave frequency of 0.1 Hz. The encounter frequency for head seas falls in the same frequency band as that of the natural pitch frequency of the patrol boat. It has not been possible to separate the two frequency components in the case of the head sea runs in the same way as for the following sea runs.

The pitch per unit waveheight is shown plotted in figure 52 which shows that the pitch is reasonably constant as the r.p.m. increase from 600 to 1250 in the case of beam and following seas. The head sea pitch shows some evidence of a minimum at 100 r.p.m.

5.3.3 Vertical Acceleration. The vertical acceleration spectra measured at the two locations (for'd and CO's cabin) are shown in figures 53 to 60. The time series records of vertical acceleration (figures 30 to 40) show that the acceleration at the two locations were very similar and were in phase and that the acceleration at the for'd position was greater than that at the CO's cabin position.

All spectra show a peak at a frequency of 0.35 Hz and the level of this peak is greatest for runs into a head sea. Many of the runs also show another peak at a frequency of 0.8 Hz. The amplitude of this peak however is generally some 20 dB below the peak at 0.35 Hz.

The vertical acceleration per unit waveheight is shown in figure 61. When the patrol boat is running into a head sea the vertical acceleration at both locations increases with r.p.m. In the case of the following sea there is a minimum at 1000 r.p.m. When the sea is on the beam the vertical accelerations at 600 r.p.m. and 1000 r.p.m. are about the same. At 1250 r.p.m. there is an increase in vertical acceleration. In general the vertical acceleration per unit waveheight in the CO's cabin was between 55% and 70% of that in the for'd position.

5.3.4 Lateral Acceleration. The lateral acceleration spectra measured in the CO's cabin and in the wheelhouse are shown in figures 62 to 69. Most spectra show a peak at a frequency of about 0.17 Hz, which is the same as that shown in the roll spectra (figures 41 to 44). Many of the spectra measured with a following sea show peaks at frequencies between 0.04 Hz and 0.08 Hz. Peaks in this frequency range were shown in section 5.3.2 to arise from the encounter frequency. If the sea were unidirectional then components at the encounter frequency would appear only in the cases of the following and head seas and not for beam seas. However the real sea is not unidirectional and whilst most of the frequency components propagate in the direction of the prevailing sea there are significant amounts of energy at these frequencies propagating in other directions. It is this which causes energy at the encounter frequency to appear when runs are carried out in beam seas. The lateral acceleration spectral peak at about 0.17 Hz is greatest for the beam sea runs.

An inspection of the lateral acceleration plots at the two locations (figures 32 to 40) shows that the acceleration at the wheelhouse is greater than at the CO's cabin. Another feature is that the two accelerations are 180^{o} out of phase ie when the one acceleration is positive the other is negative. This shows that roll is occurring about a point somewhere between the two locations.

The lateral acceleration per unit waveheight is plotted in figure 70. In the case of the beam sea the lateral acceleration in the CO's cabin position is fairly constant as r.p.m. increases. The lateral acceleration in the wheelhouse is almost twice that in the CO's cabin at 600 r.p.m. and the difference between the acceleration at the two positions falls rapidly as the r.p.m. increases.

With a following sea the lateral acceleration generally increases with r.p.m. The head sea case however shows a minimum at 1000 r.p.m.

5.3.5 Rate of Change of Vertical Acceleration

The vertical acceleration measured at the for'd position and its derivative is shown in figures 71 to 76. The derivative was obtained with an operational amplifier differentiator as described in section 4.3 .

The spectra of the rate of change of vertical acceleration at the for'd position which were obtained by the transformation method described in section 4.3 are shown in figures 77 to 80. All spectra show a peak in energy at 0.35 Hz. At lower frequencies there is no significant energy as the spectral level falls rapidly. The level of the peak at 0.8 Hz is increased relative to the main peak because of the f^2 term in equation (1) .

The rate of change of vertical acceleration (measured at the for'd position) per unit waveheight is shown in figure 81. In the case of the head sea the derivative of vertical acceleration increases rapidly as the r.p.m. increases, whilst a following sea gives a slight minimum at 1000 r.p.m. For a beam sea the vertical acceleration derivative increases as r.p.m. increases.

5.3.6 Transfer Function. The transfer functions for roll and pitch were derived using the methods described in section 4.4 and the resulting transfer functions are shown plotted in figures 82 and 83. The lower frequency limit of the transfer functions is 0.03 Hz because the waveheight spectra could not be measured below this frequency owing to the low frequency response of the waverider buoy (see section 2.2) .

The roll transfer function has two peaks, one at 0.17 Hz probably a fundamental property of the hull and another at a lower frequency which would be related to the encounter frequency. Unfortunately the lack of low frequency response does not allow the frequency of this peak to be accurately determined.

The pitch transfer function has similar features in that the peak at 0.35 Hz is fairly constant as the r.p.m. and heading varies showing that this frequency is a property of the hull. The frequency of the remaining peak varies with r.p.m. and is related to the encounter frequency.

Similarly the transfer functions could be obtained for the lateral and vertical accelerations.

6. CONCLUSIONS

In this report data are presented on roll, pitch, vertical and lateral accelerations of the Attack Class Patrol Boat HMAS BOMBARD. These data were obtained during sea keeping trials carried out off Sydney during October 1976. Measurements of waveheight during the trial were carried out using a Datawell Waverider Buoy that was moored in the trials area. Accurate navigation was obtained from the Miniranger III System with transponders located at Barranjoey Head Light and at Macquarie Light. The patrol boat carried out runs at 3 different speeds and with 3 different headings to the prevailing sea (head, beam and following).

The roll, pitch and vertical and lateral acceleration and waveheight data were processed to obtain energy spectra. It was shown that because of the predominantly low frequency components present it was necessary to take record lengths of at least 20 minutes to obtain reasonable estimates of the spectra.

A method of obtaining the spectrum of the rate of change of a function given the spectrum of that function has been derived. The required spectrum is obtained by a straightforward transformation of the original spectrum. The transformation can be done graphically if desired. This technique has been used to derive the spectra of the rate of change of vertical acceleration which is an important parameter in studies of operator motion sickness.

The root-mean-square values of roll, pitch, vertical and lateral acceleration and the rate of change of vertical acceleration were derived by taking the square root of the area under each spectral curve.

During the trials period the waveheight varied between 0.61 m (r.m.s.) and 0.96 m (r.m.s.). It was found that the Pierson-Moskowitz waveheight spectra agreed quite well with the spectra measured during the trial.

All the roll spectra showed a peak at a frequency of 0.17 Hz (ie a roll period of 6 seconds) The plots of roll per unit waveheight showed that roll was greatest for beam seas and that the roll steadily increased as r.p.m. increased for the case of beam and following sea runs. For a head sea run the roll per unit waveheight shows a minimum at 1000 r.p.m.

The pitch spectra show 2 distinct peaks, the upper one at 0.25 Hz due to the natural pitch frequency and a lower frequency peak which arises from the rate at which waves are encountered. It is shown that the encounter frequency is given by the rate at which successive wave crests are met. The agreement between the calculated frequency and the measured frequency as the patrol boat speed was varied in the case of an overtaking sea was very good. The pitch per unit waveheight is greater for a head sea than for beam and following seas. It is also constant at about 3.6 r.m.s. degrees/r.m.s. metre for beam and following seas. In the case of a head sea there is evidence of a minimum at 1000 r.p.m.

It was found that the vertical accelerations measured in the for'd position were greater than those measured in the CO's cabin. Typically vertical accelerations of up to ±0.6 g peak were measured in the for'd position when the patrol boat was moving into a head sea (r.m.s. waveheight of 0.85 m) at 15 knots. The peak vertical acceleration was found to be at a frequency of 0.35 Hz. When the patrol boat is running into a head sea the vertical acceleration increases rapidly with r.p.m.

The lateral acceleration gives a spectral peak at 0.17 Hz which is the same as that shown by the roll spectra. It is shown that the lateral acceleration measured at the wheelhouse is greater than that at the CO's cabin and that the two accelerations are 180° out of phase. This indicates that the center of roll is between the two positions. The lateral acceleration per unit waveheight at the wheelhouse decreases with r.p.m. whereas at the CO's cabin position it is relatively constant.

The spectra of the rate of change of vertical acceleration show a strong peak at 0.35 Hz. The energy at lower frequencies falls off rapidly. When the patrol boat is moving into a head sea the normalised derivative of vertical acceleration increases rapidly with r.p.m.

The concept of the patrol boat as a system with an input and output which are related by a transfer function is introduced. The input is taken to be the waveheight spectrum and the output is the spectrum of roll, pitch or acceleration. Once the transfer function for a given r.p.m. and heading to the prevailing sea has been obtained the output spectrum and thus the r.m.s. value for a given waveheight spectrum can readily be obtained.

Acknowledgements

The measurements reported in this document required the assistance and co-operation of many different organizations and the author would like to thank:

1. The Commanding Officer of HMAS BOMBARD.
2. Mr. E. Tremayne of the Aeronautical Research Laboratory, Melbourne who installed the instrumentation aboard the patrol boat and operated it during the trial.

3. Mr M. Smith from the Directorate of Forward Design, Navy Office who acted as Trials Officer.

4. The Commanding Officer of HMAS WATSON for his assistance with the setting up of the Waverider receiving station.

5. The Lighthouse Section of the Department of Transport for assistance in setting up the Miniranger Stations.

6. Staff of Maritime Services Board Signal Station at South Head who supplied wind velocity measurements.

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ANNEX A

CALIBRATION DETAILS - ROLL, PITCH AND ACCELERATION SPECTRA

1. INTRODUCTION

This annex describes the method used to analyse the time series data representing the roll, pitch and acceleration and presents in some detail the calculations necessary to obtain calibrated energy spectra.

2. ROLL

The instrumentation used to obtain the roll spectra is shown in figure A1. The output from the roll gyro (of sensitivity 0.37 volts/degree) is recorded on a tape recorder. During replay the signal was low-pass filtered (f_{co} = 4 Hz) and the power spectrum was obtained using the SD 360 Digital Signal Processor. The SD 360 has a variable attenuator at the input. The tape recorder gain was set up so that 1 volt input gave 1 volt output ie 0 dB gain.

A sinusoidal calibration signal of 0.1 volt (r.m.s.) was then applied to the input of the low-pass filter. This calibration signal was equivalent to a roll of

$$\frac{0.1}{0.037} = 2.7 \text{ degrees (r.m.s.)}$$

which corresponds to a power spectral density of

$$20 \log 2.7 = 8.6 \text{ dB re 1 degree.}$$

During this calibration the input attenuator was set at 30 dB. Therefore with 0 dB attenuation the calibration signal corresponds to

$$8.6 - 30 = -21.4 \text{ dB re 1 degree.}$$

Since the analysis bandwidth was 0.01 Hz the spectral levels must be normalised to a 1 Hz bandwidth by subtracting

$$10 \log 0.01 = -20 \text{ dB (ie adding 20 dB).}$$

Thus the calibration signal with 0 dB attenuation corresponds to a power spectral density of - 21.4 + 20 =-1.4 dB re 1 degree/Hz, which is equivalent to 0.72 degree^2/Hz since 10 log 0.72 = -1.4 .

If the SD 360 attenuator is set to 10 dB during analysis then the calibration signal corresponds to

- 1.4 + 10 = + 8.6 dB re 1 degree/Hz (ie 7.24 degree^2/Hz) .

Similarly with 20 dB attenuation, a level of - 1.4 + 20 = +18.6 dB re 1 degree/Hz (ie 72.44 degree^2/Hz) is obtained.

In the figures in this report which show roll spectra the units on the ordinate are degree^2/Hz. The conversion between dB re 1 degree/Hz and degree^2/Hz is given by

(dB re 1 degree/Hz) = 10 log (degree^2/Hz)

3. PITCH

A similar technique was used to obtain pitch spectra. In this case the transducer had a sensitivity of 0.1 volts/degree. The calibration signal was equivalent to a pitch of 1 degree (r.m.s.) . It follows that for 0 dB attenuation on the SD 360 the calibration signal corresponds to a power spectral density of - 10 dB re 1 degree/Hz (ie 0.1 degree^2/Hz) . An attenuation level of 10 dB gives 0 dB re 1 degree/Hz (ie 1.0 degree^2/Hz) and 20 dB to 10 dB re 1 degree/Hz (ie 10 degree^2/Hz).

4. ACCELERATION

The spectra of vertical and lateral acceleration were similarly derived. The sensitivity of the accelerometers was 1.4 volts/g.

ANNEX B

CALIBRATION DETAILS - WAVEHEIGHT SPECTRA

The instrumentation used at HMAS WATSON to measure the waveheight spectra is shown in figure B1. Using similar reasoning to that given in Annex A the waveheight spectra can be obtained as follows.

The overall sensitivity of the buoy-receiver system was 1 volt/metre and the calibration signal corresponds to a power spectral density of

$20 \log 1 = 0$ dB re 1 m in a 0.01 Hz bandwidth which is equal to + 20 dB re 1 m/Hz . Thus if the SD 360 input attenuation is fixed during the calibration and measurement phase then the calibration signal corresponds to

$$20 \text{ dB re } 1 \text{ m/Hz} \quad (\text{ie } 100 \text{ m}^2/\text{Hz}) .$$

ANNEX C

THE POWER SPECTRUM OF THE DERIVATIVE OF A FUNCTION

The Fourier transform $X(f, T)$ of a function $x(t)$ is given by

$$X(f, T) = \int_{0}^{T} x(t)\, e^{-j2\pi ft}\, dt \qquad \text{---} \qquad C1$$

and the inverse transform is given by

$$x(t) = \int_{-\infty}^{\infty} X(f,T)\, e^{j2\pi ft}\, df \qquad \text{---} \qquad C2$$

In order to reduce the random errors in $X(f, T)$ if $x(t)$ is a nondeterministic time series individual estimates are computed from N sample records. At each frequency an average over these N samples is then taken ie if the power spectrum $G_x(f)$ is obtained by averaging over N spectra then :

$$G_x(f) = \frac{1}{N} \sum_{i=1}^{N} X_i(f,T)\, X_i^*(f,T) \qquad \text{---} \qquad C3$$

where X^* is the complex conjugate.

Differentiating C2 with respect to time,

$$\frac{dx(t)}{dt} = j2\pi f \int_{-\infty}^{\infty} X(f,T)\, e^{j2\pi ft}\, df \qquad \text{---} \qquad C4$$

$$\text{ie} \quad \dot{x}(t) = j2\pi f\, x(t) \qquad \text{---} \qquad C5$$

The Fourier transform $Y(f,T)$ of the derivative of $x(t)$ is given by

$$Y(f,T) = \int_{0}^{T} \dot{x}(t)\, e^{-j2\pi ft}\, dt$$

$$= j2\pi f \int_0^T x(t)\, e^{-j2\pi ft}\, dt \qquad \text{--- C7}$$

$$= j2\pi f\, X(f,T) \qquad \text{--- C8}$$

The power spectrum $G_{\dot{x}}(f)$ of $\dot{x}(t)$ is given by

$$G_{\dot{x}}(f) = \frac{1}{N} \sum_{i=1}^{N} Y_i(f,T)\, Y_i^*(f,T) \qquad \text{--- C9}$$

Substituting C8 into C9

$$G_{\dot{x}}(f) = 4\pi^2 f^2\, G_x(f) \qquad \text{--- C10}$$

Thus if the power spectrum of a function is known then the power spectrum of its derivative can be obtained by multiplying the power at a given frequency f by the factor $4\pi f^2$. At a frequency of 0.16 Hz $4\pi f^2 = 1$ and the two spectra are equal.

ANNEX D

THE PEIRSON-MOSKOWITZ WAVEHEIGHT SPECTRUM

The Pierson-Moskowitz waveheight spectrum (Pierson and Moskowitz 1964) has been found in the literature to adequately describe "fully developed" seas. This is a one-parameter distribution which relates wind velocity to the spectrum which results when equilibrium is established.

The spectral form is given by

$$\phi(h;\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[- \gamma \left(\frac{\omega_o}{\omega} \right)^4 \right] \qquad \text{--- D1}$$

where $\phi(h;\omega)$ = waveheight spectrum as a function of angular frequency.

$\alpha = 8.10 \times 10^{-3}$

$\gamma = 0.74$

$\omega_o = g\, U_{19.5}^{-1}$

$U_{19.5}$ = wind velocity 19.5 m above the mean sea level (m/sec)

the units of $\phi(h;\omega)$ are m^2/Hz .

Equation D1 is more convenient if the variable ω is changed to frequency f . Since the area under the two spectral forms is given by H_{rms}^2 then

$$\int_0^\infty \phi(h;\omega)d\omega = \int_0^\infty \phi(h;f)\, df = H_{rms}^2$$

and since $\omega = 2\pi f$ it follows that

$$\phi(h;f) = 2\pi\, \phi(h;\omega)$$

and equation D1 can be written as

$$\phi(h;f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left[-\gamma\left(\frac{g}{U_{19.5}\, 2\pi f}\right)^4\right] \qquad \text{--- D2}$$

Equation D2 has been plotted in figure D1 to show the Pierson-Moskowitz spectrum for several different wind speeds. It can be seen that as the wind speed increases the energy of the waves increases and the frequency at which most of the wave energy is concentrated decreases.

Also plotted on figure D1 is the amplitude response of the Datawell Waverider buoy. The low frequency roll off in the buoy response occurs at a frequency below that at which there is any significant wave energy. Similarly at high frequencies there is very little energy above 1 Hz. The resonance peak at 0.8 Hz does cause a slight bump on the high frequency limiting line of the spectrum.

ANNEX E

BRIEF REPORT ON ACTIVITIES OF TRIALS WEEK

Monday 25-10-76

BOMBARD set out at 0930. The intention for the day was to lay the waverider buoy and complete some of the trial runs.

The buoy was successfully laid some 14 km off Sydney Heads. A head sea and beam sea run were attempted, however the CO considered the weather too rough and so BOMBARD returned to WATERHEN.

The recordings for these two runs were unsuccessful as it seems the tape recorder was not set up correctly.

Tuesday 26-10-76

Approx Drafts: fwd - 6'0"
aft - 7'7" (port)

BOMBARD set out at 0845.

Two runs were successfully completed and recorded. These were:

Head sea: 0950 - 1015 rpm 1000

Following sea: 1020 - 1045 rpm 1000

Ship motion questionaires were completed for these runs.

The sea consisted of a SE swell with a NE wind. However, the sea was essentially unidirectional. Again, the CO considered the weather too rough and so the ship returned.

In the afternoon two rolling tests were carried out 1315 - 1325. The results were recorded on magnetic tape. This was processed by RANRL and a graphical output obtained.

Since the weather was most favourable it was hoped to conduct the inclining experiments in the afternoon. However the crane proved to be not available due to water contamination of the fuel. Before it was realized that the inclining would need to be postponed, accurate draft readings were taken:

Fwd:		5'	$11\frac{3}{4}$"
Aft:	Stbd	7'	10"
	Port	7'	8¼"

Wednesday 27-10-76

Approx Drafts: fwd - 5'11"
aft - 7' 6" (port)

BOMBARD set out at 0830.

The sea was more moderate than the previous two days. It appeared to consist of an easterly swell with a SE wind, but essentially unidirectional.

Observed wave height was approx 5 ft

Observed wave period was approx 5 sec.

All nine runs were completed and recorded successfully, these were as follows:

Run 1: Head Sea: 0950 - 1016, 1250 r.p.m

- some strong lateral accelerations were experienced.
- run out short due to loss of contact with transponder B of Miniranger (out of range).

Run 2: Following Sea: 1022 - 1052, 1250 r.p.m

Run 3: Beam Sea: 1054 - 1123, 1250 r.p.m - sea on port side

- lost contact with transponder A at 1106, run continued, assume speed over remainder of run constant.

Run 4: Beam Sea: 1134 - 1204, 600 r.p.m - sea on starboard side

Run 5: Head Sea: 1207 - 1237, 600 r.p.m

Run 6: Following Sea: 1240 - 1309, 600 r.p.m

- A fresh tape was loaded after run 6.

Run 7: Beam Sea: 1319 - 1349, 1000 r.p.m

- Sea on starboard side.

Wednesday, 27-10-76 (Cont'd)

Run 8: Head Sea: 1350 - 1420, 1000 r.p.m

- E swell still predominant, however southerly wind increasing in effect.
- Early in run it was necessary to bring course 20° further south to encounter predominant wave train as head sea.

Run 9 Following Sea: 1422 - 1452; 1000 r.p.m

Ship motion questionnaires were completed by the crew for this day.

Thursday, 28-10-76

Approx Drafts: fwd 5'9"
aft 7'9" port

With weights on and after moving ship to side of wharf, approx drafts:

fwd: 5'9"
aft: 8'1" stbd

Some delay was experienced due to the crane being used by clearance divers to recover submerged fenders alongside the wharf.

Three weight shifts were completed before lunch and three after. Drafts were then taken.

A reference square was taped to the port side of the hull amidships for the following day's photographic runs (3' x 2')

It was considered that the weather was unfavourable for the performance of an accurate inclining experiment.

Friday 29-10-76

A photographic record was obtained of BOMBARD making three speeds, corresponding to 600, 1000, 1250 r.p.m . A still water photograph was also taken. From these it is hoped to determine the running waterline at these speeds, and so model the submerged hull more accurately.

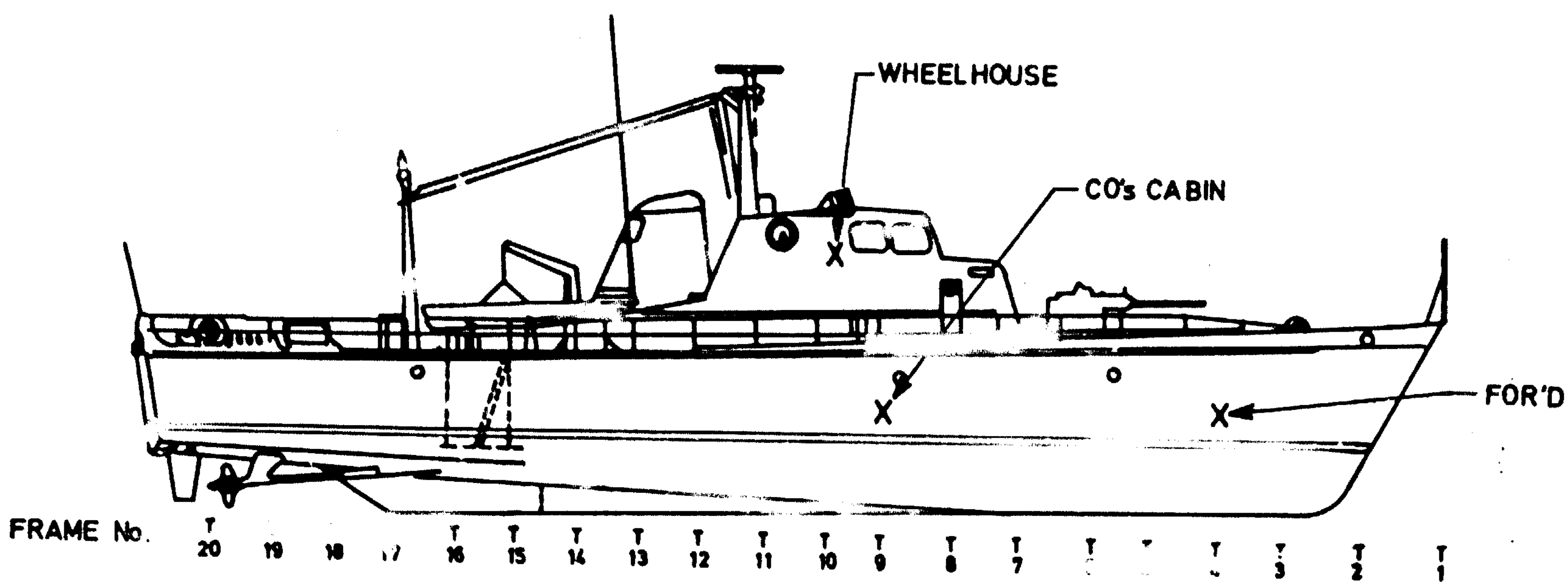
DETERMINATION OF TRANS RADIUS OF GYRATION

From graphical record of rolling tests the period of roll is 5.66 sec.

Now, $k = \dfrac{T\sqrt{GM}}{1.108}$ $T = 5.66$ sec

$GM = 10.5 - 8.7$

$= 6.9$ ft. $= 1.8$ ft



WHEELHOUSE — LATERAL ACCELERATION
FOR'D — VERTICAL AND LATERAL ACCELERATION
CO's CABIN — VERTICAL AND LATERAL ACCELERATION. PITCH & ROLL
NOTE: LATERAL ACCELERATION TRANSDUCER IN FOR'D POSITION WAS UNSERVICEABLE

Fig.1 Location of transducers in Patrol Boat during sea keeping trial. October, 1976.

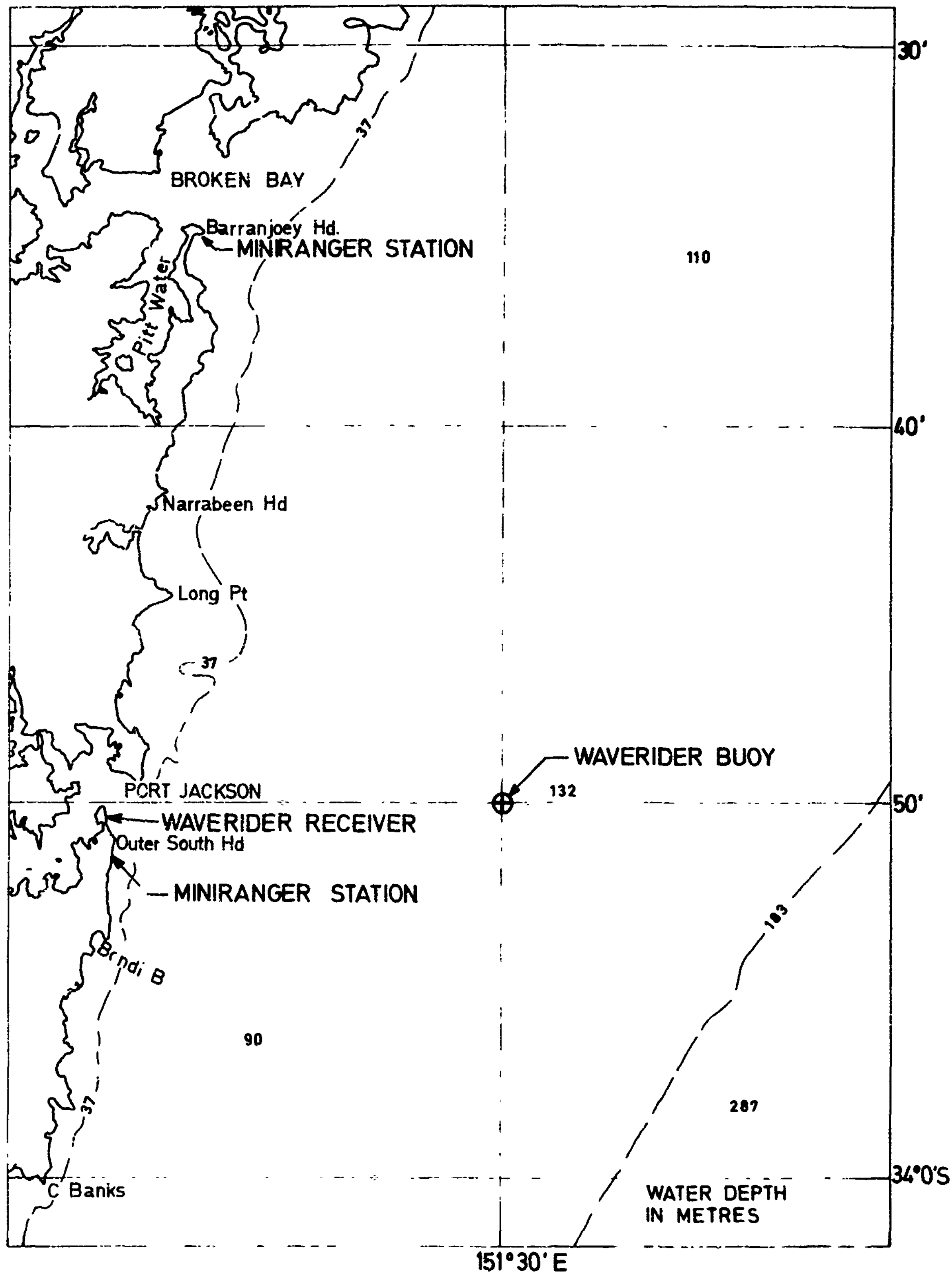


Fig 2. Location of Patrol Boat seakeeping trial. October 1976.

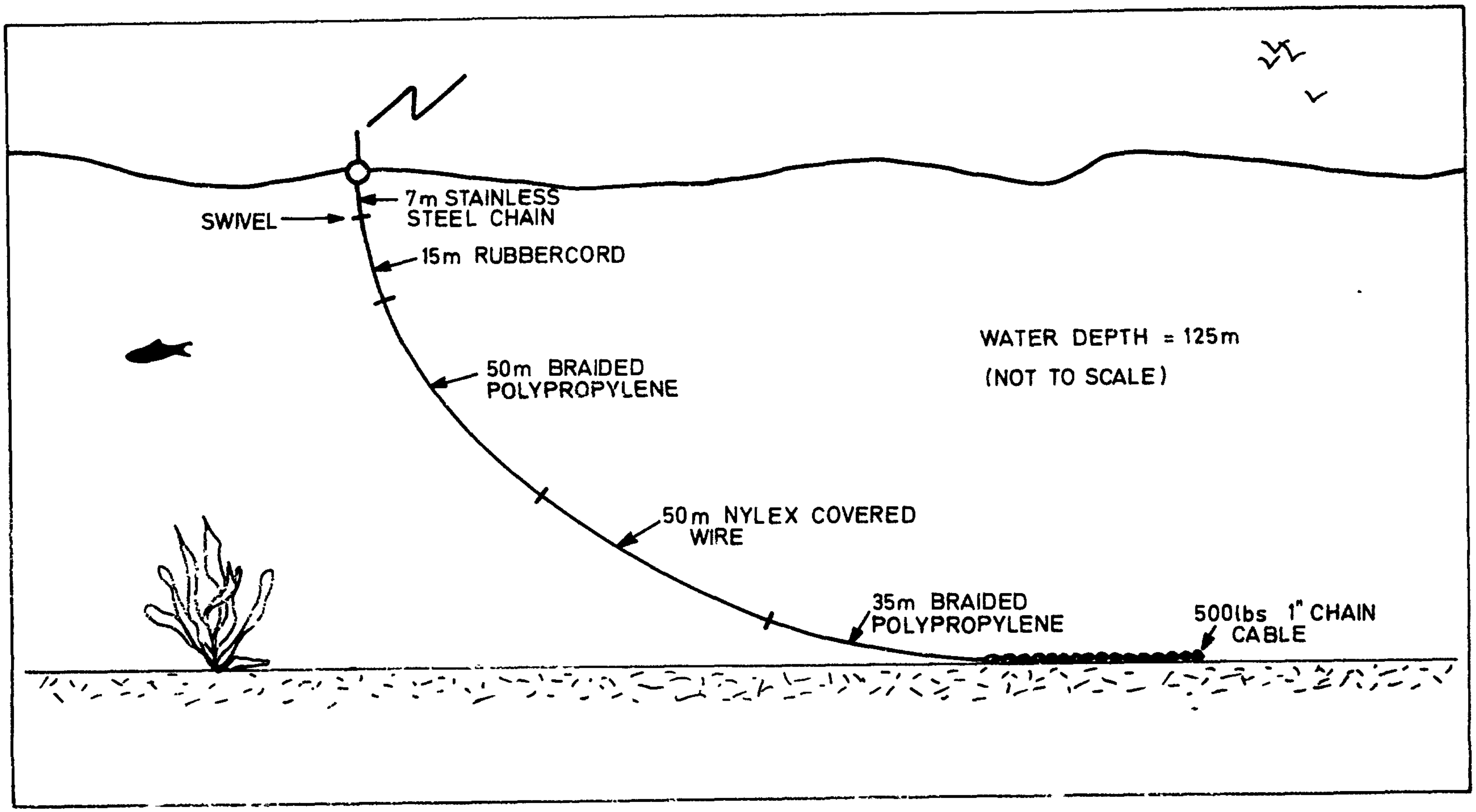


Fig. 3. Waverider buoy mooring details.

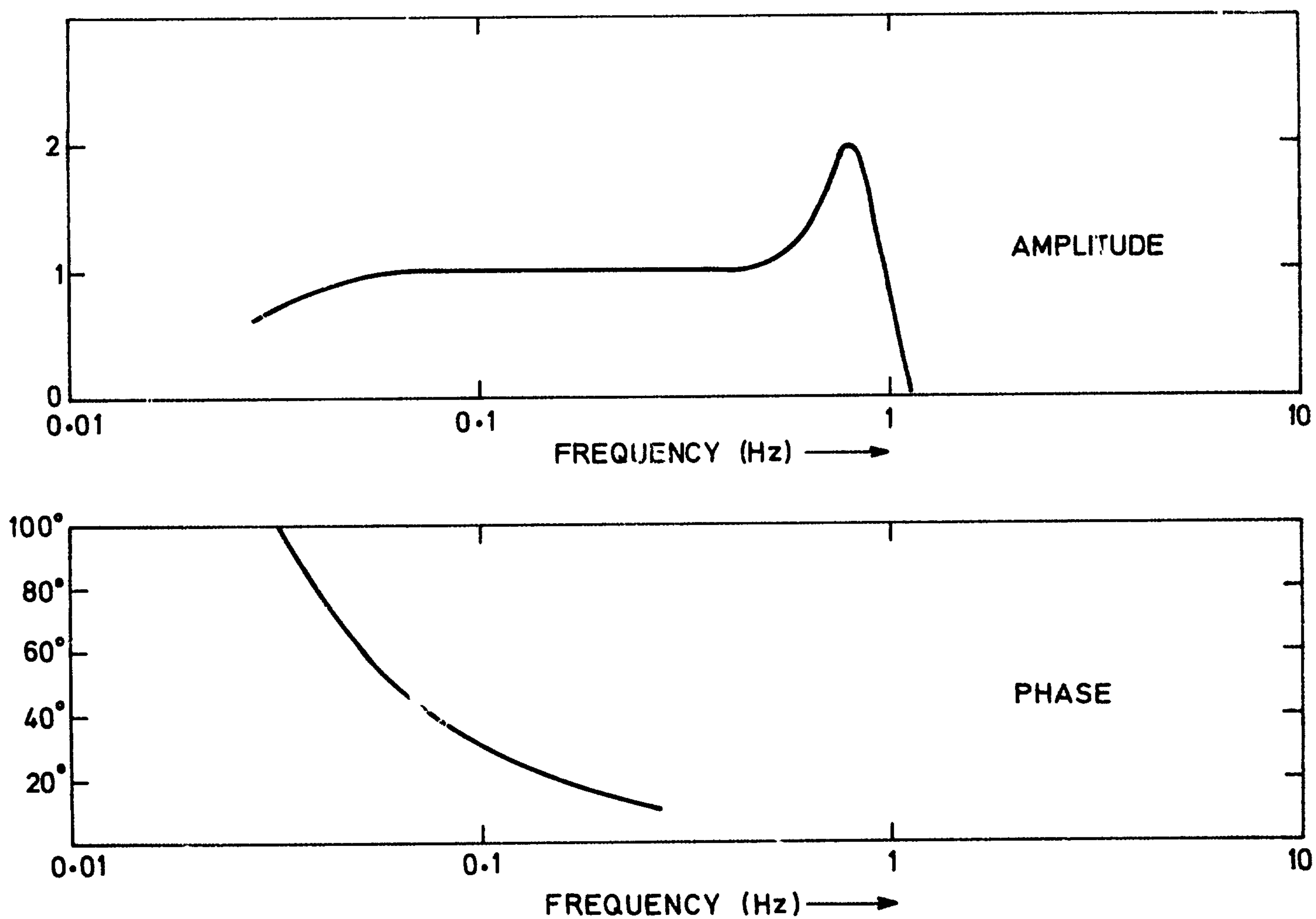


Fig. 4. Response of waverider buoy.

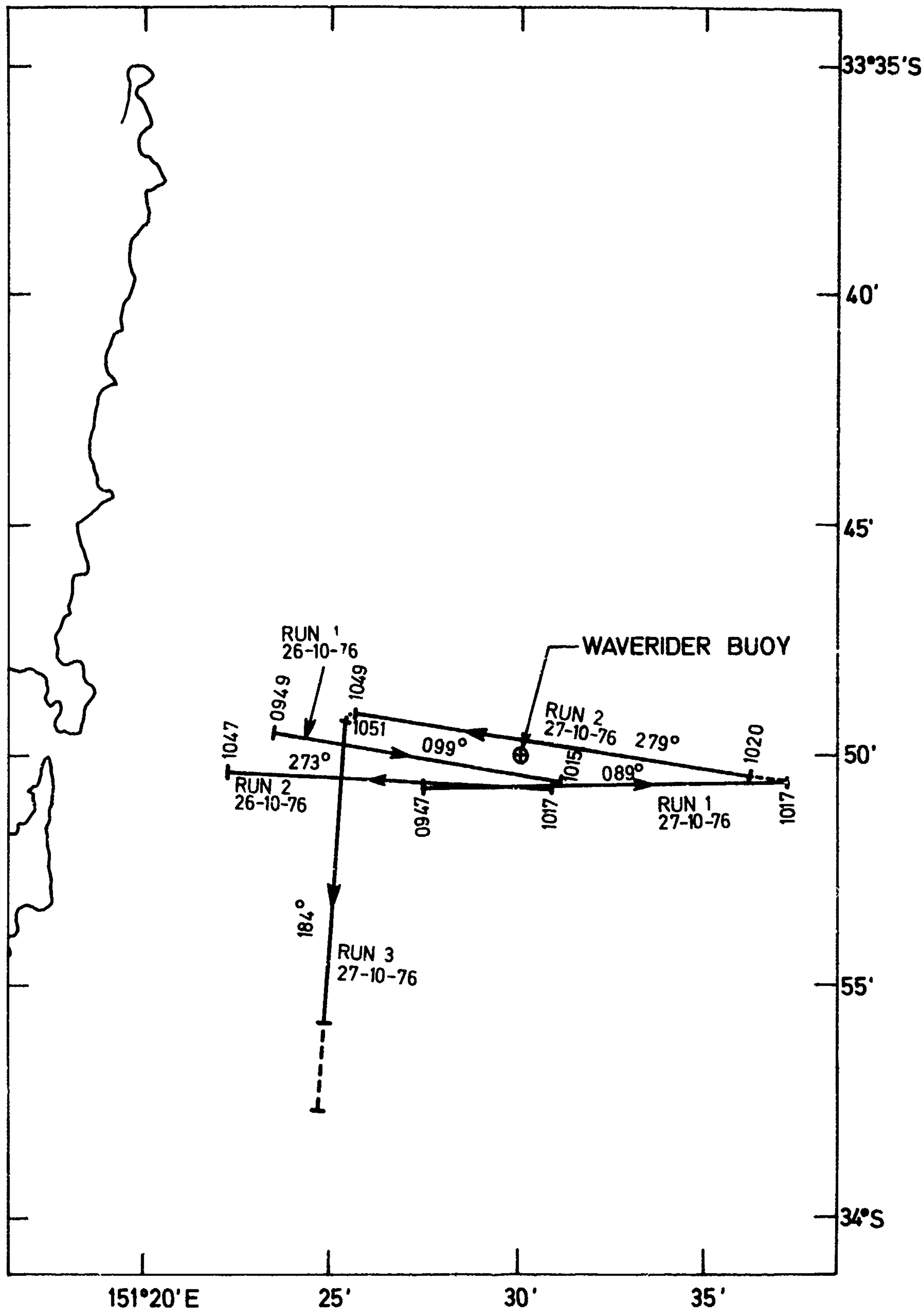


Fig. 5. Patrol Boat seakeeping trial. Plot of Runs 1&2 26 OCT 76 and Runs 1 to 3 27 OCT 76.

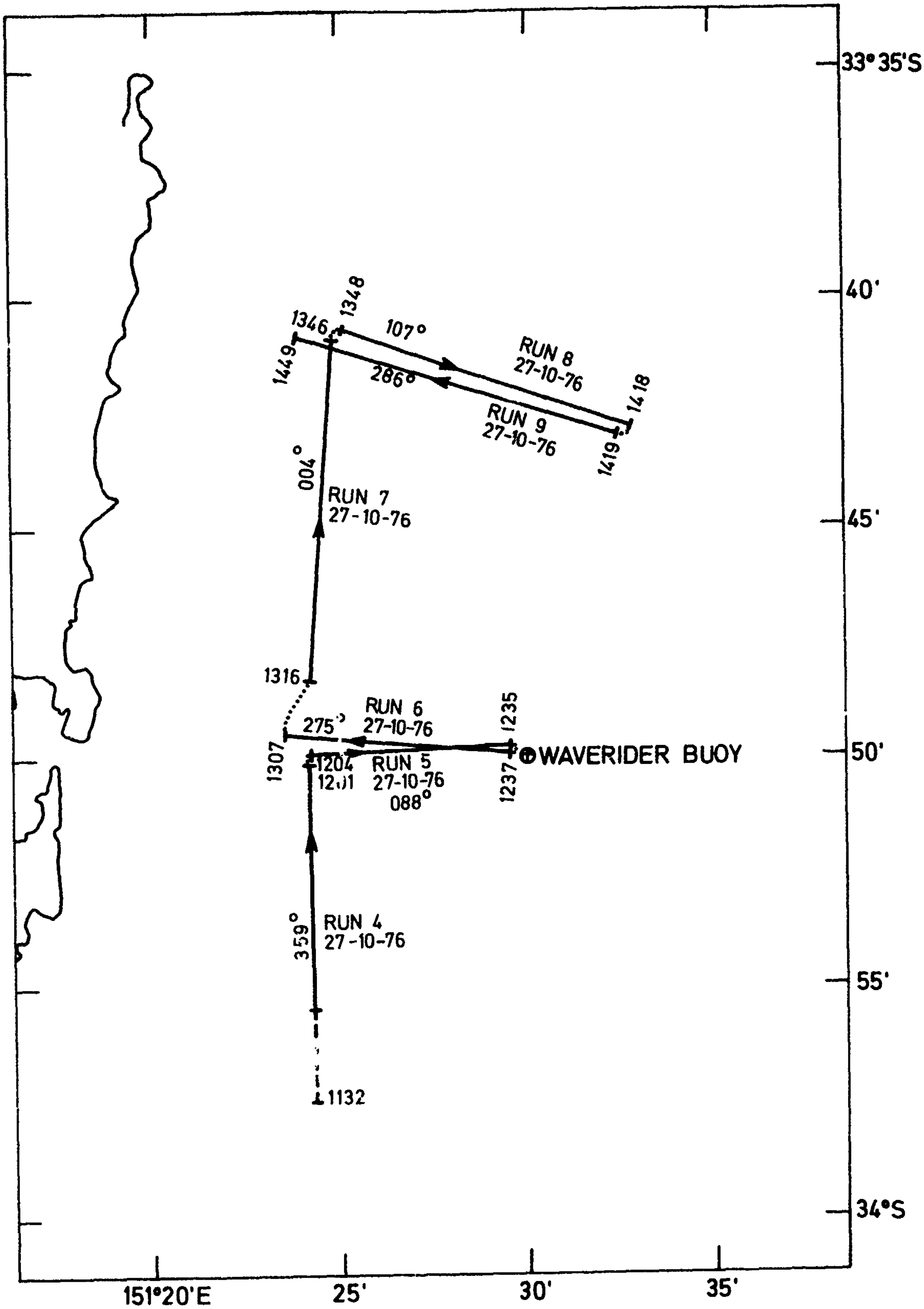


Fig. 6. Patrol Boat seakeeping trial. Plot of Runs 4 to 9 27 OCT 76.

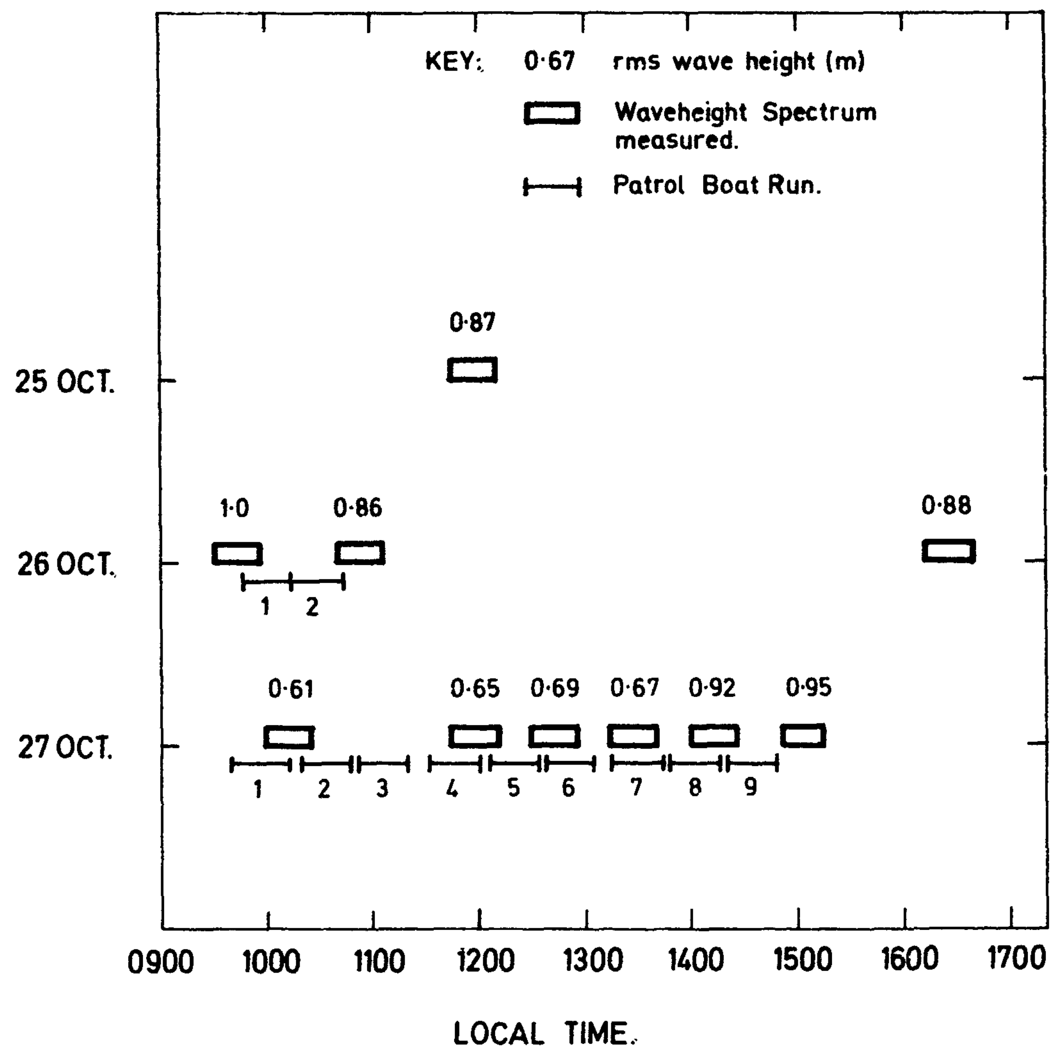


Fig. 7. Times of waveheight spectra measurement and Patrol Boat runs.

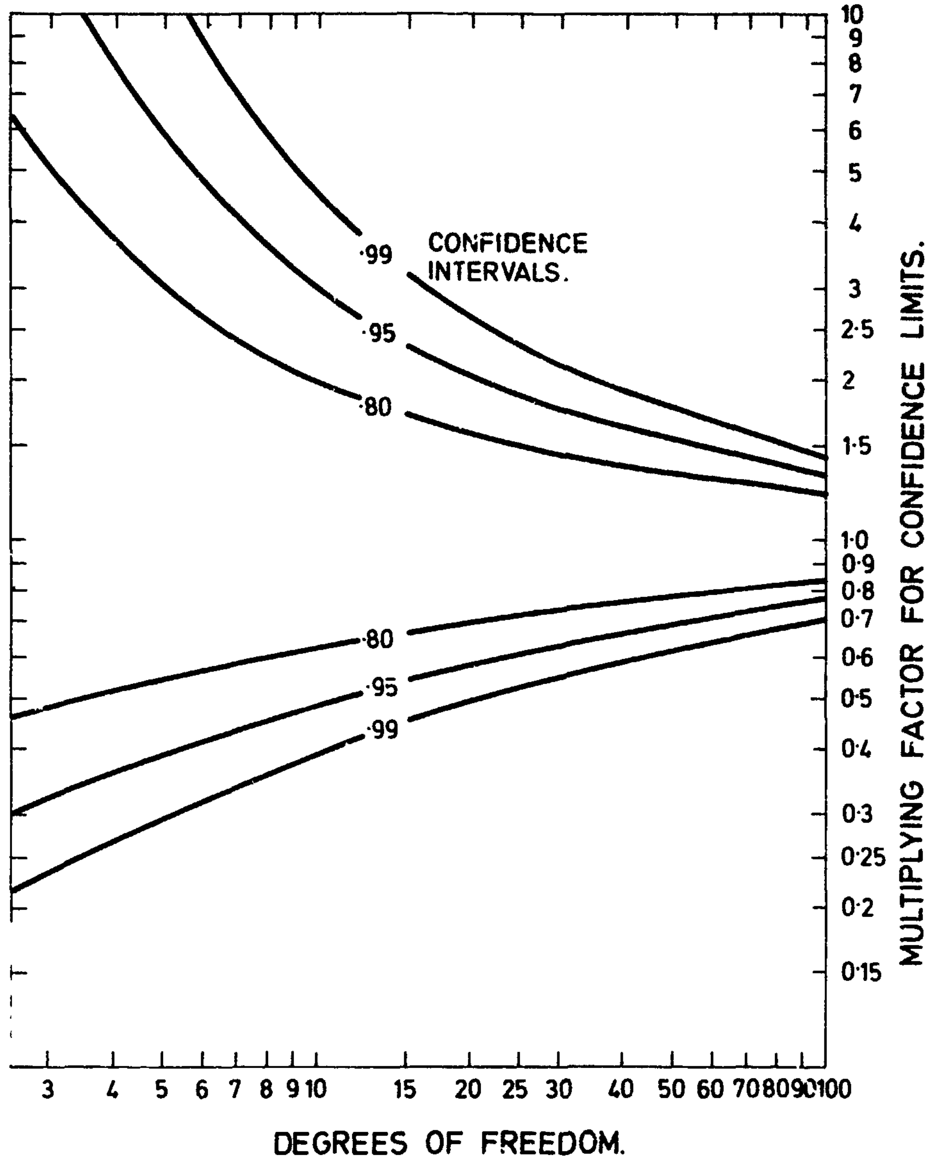


Fig. 8. Plot of confidence limits as degrees of freedom for chi-square distribution.

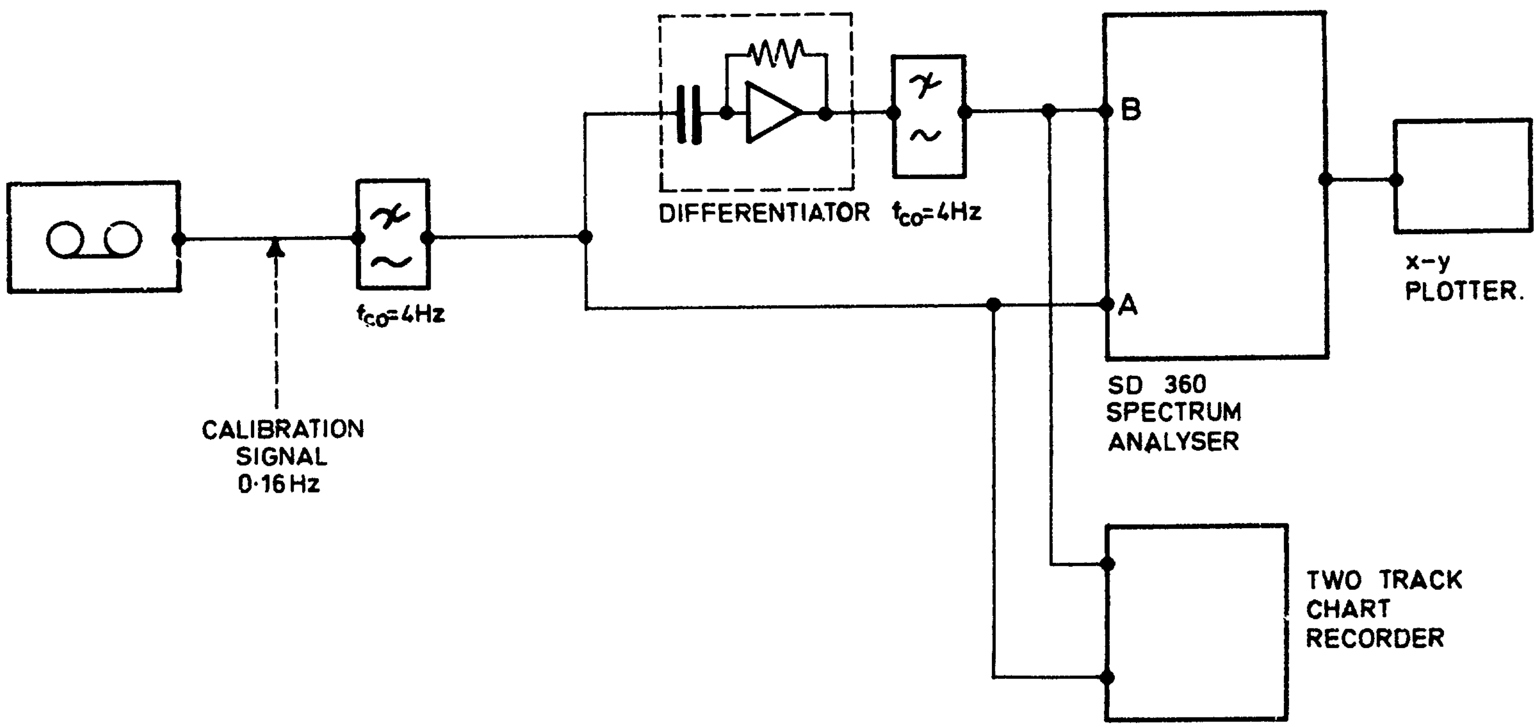


Fig. 9. Measurement of the power spectrum of the derivative of vertical acceleration using a differentiator and a spectrum analyser.

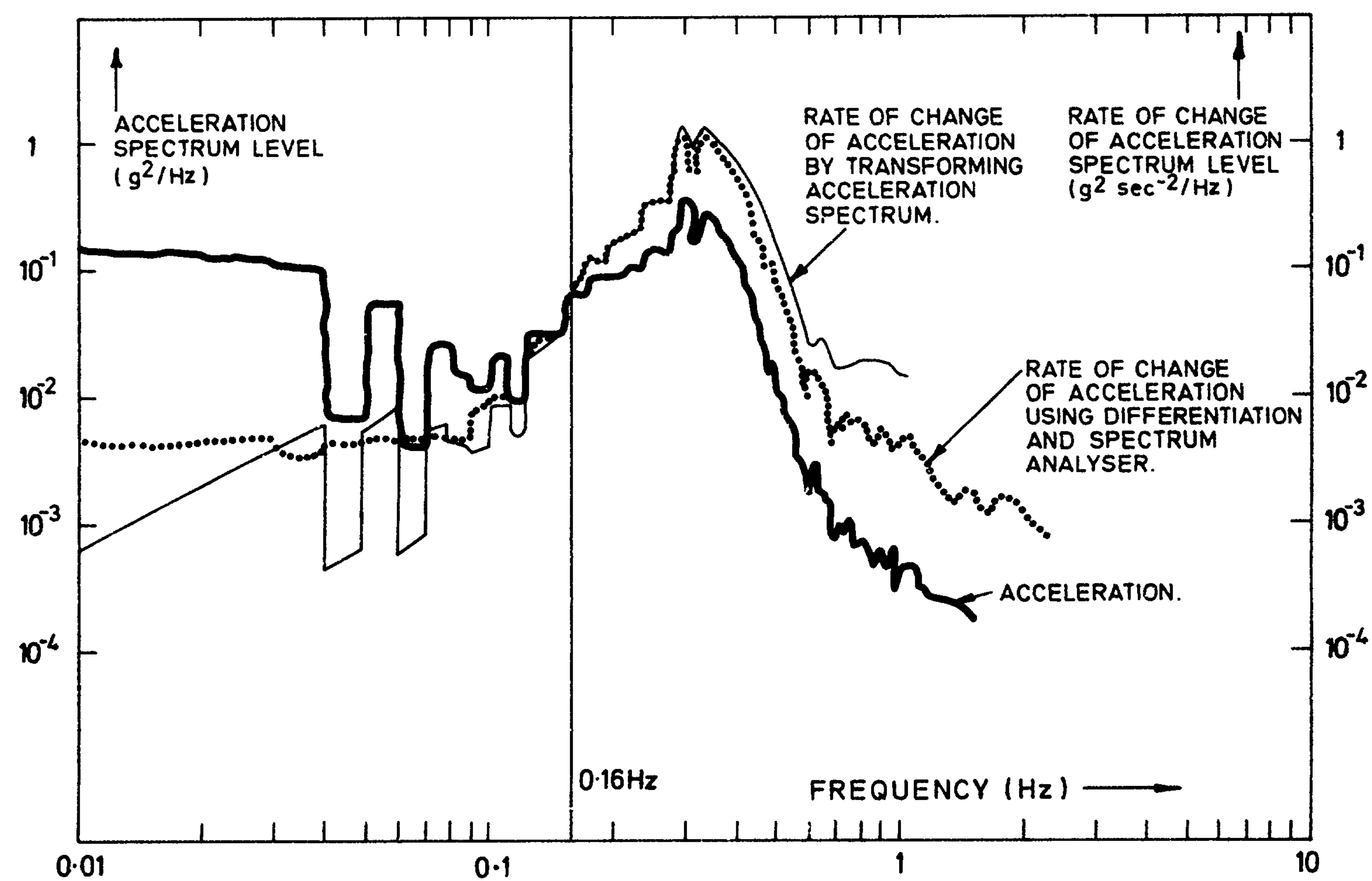


Fig.10. Typical spectra of vertical acceleration and its derivative.

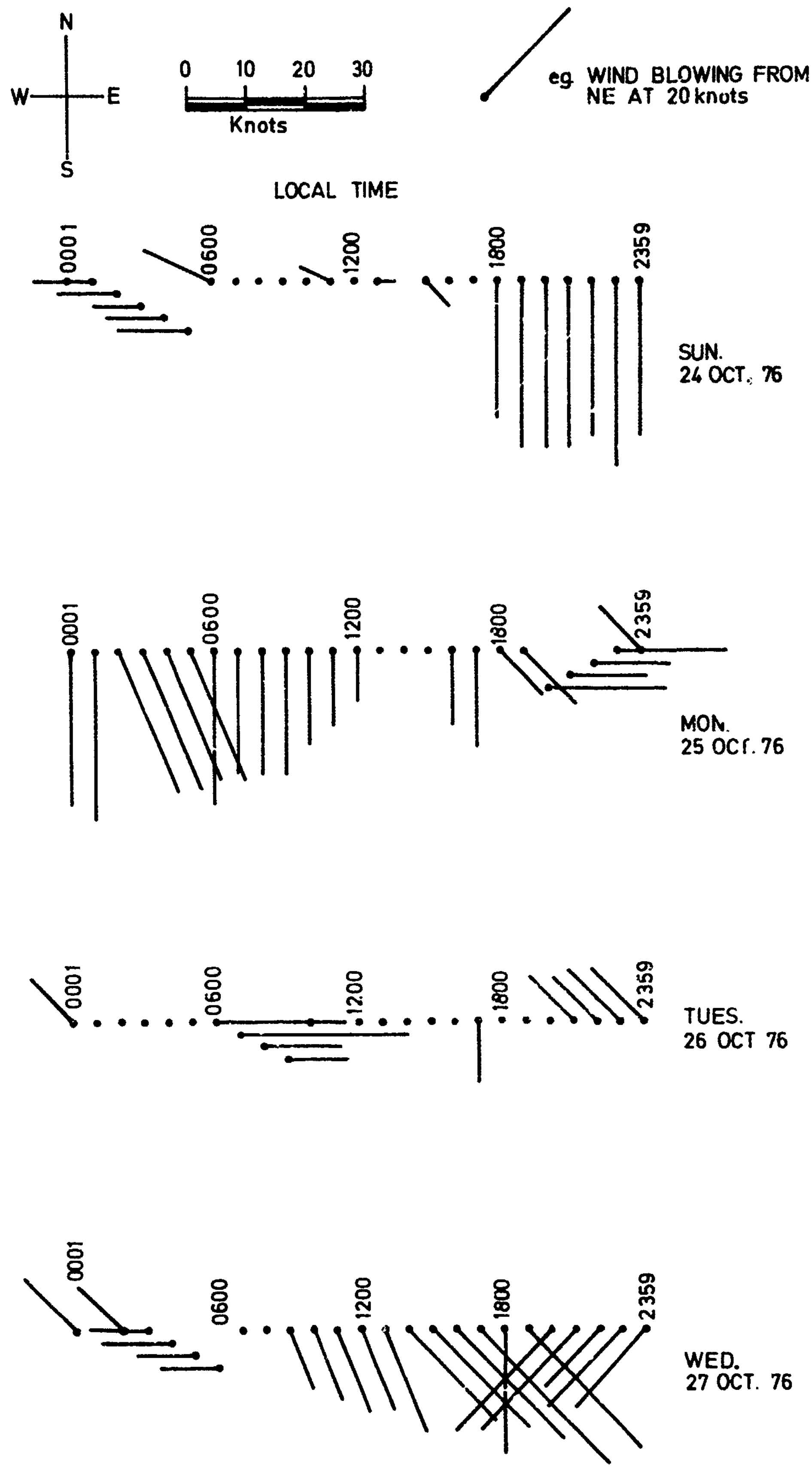


Fig.11. Wind speed and direction measured at South Head signal station 24-27 Oct.76.

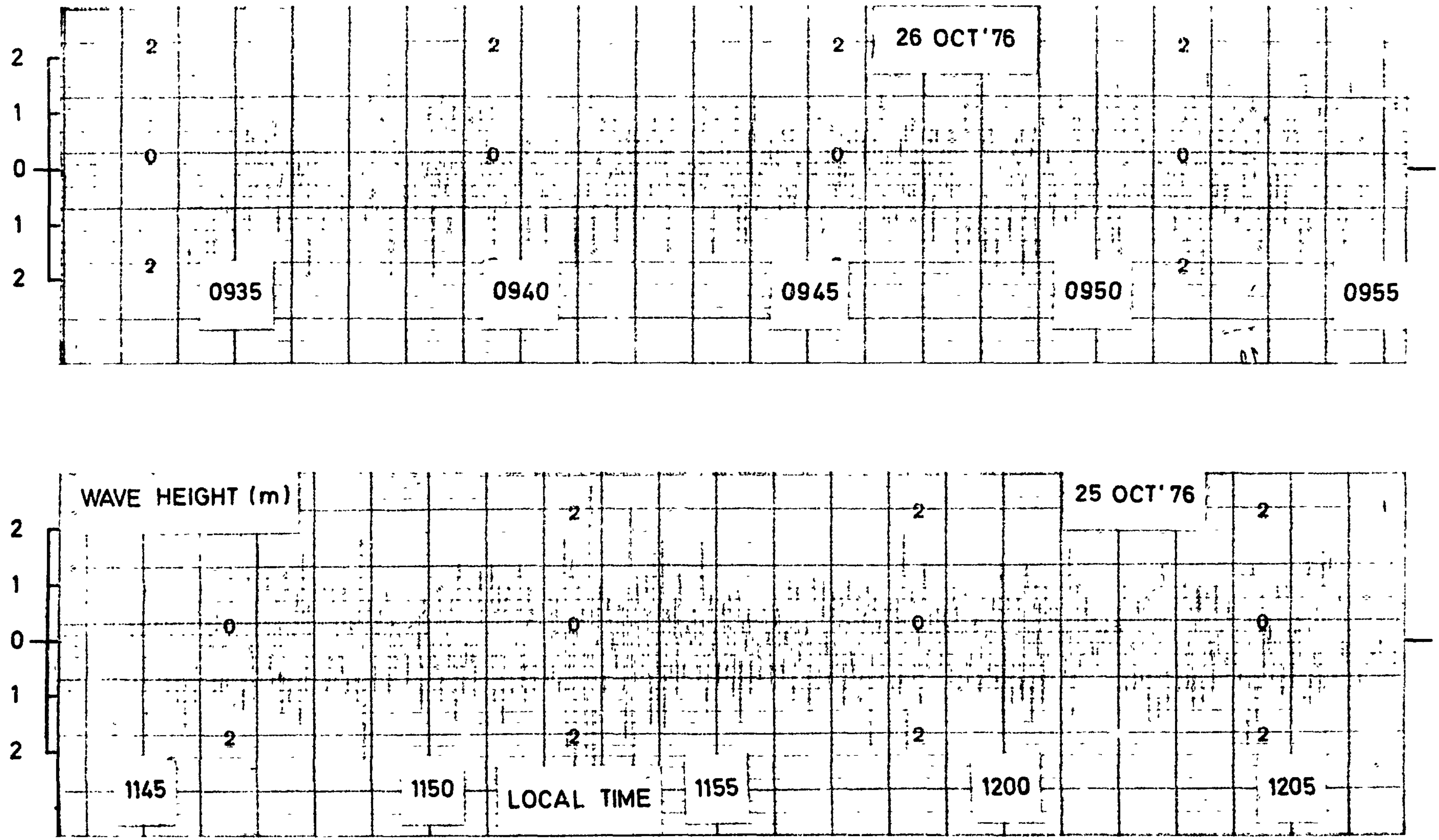


Fig.12. Wave height record 1144 25 OCT'76 and 0932 26 OCT'76.

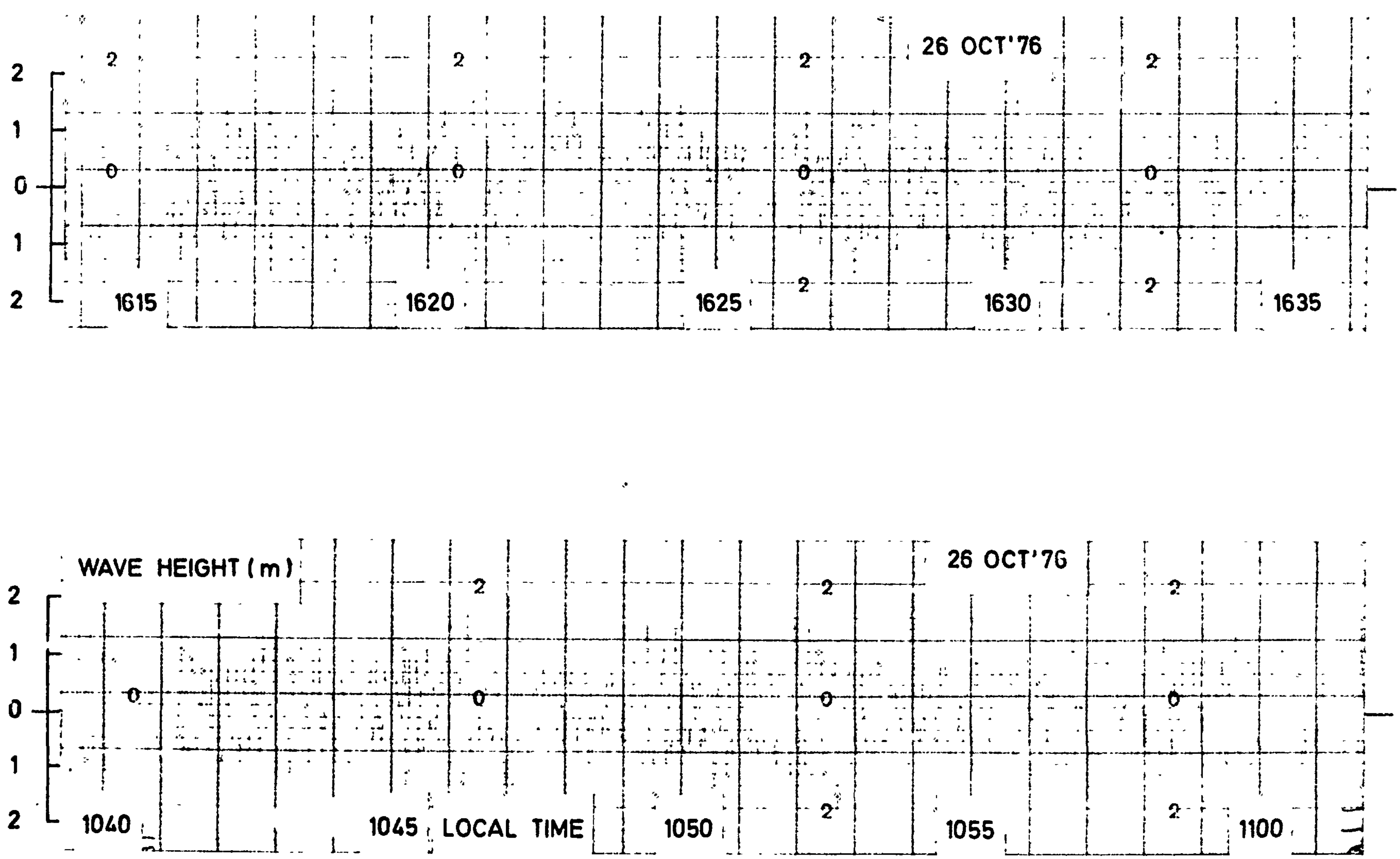


Fig.13. Wave height record 1040 26 OCT'76 and 1614 26 OCT'76.

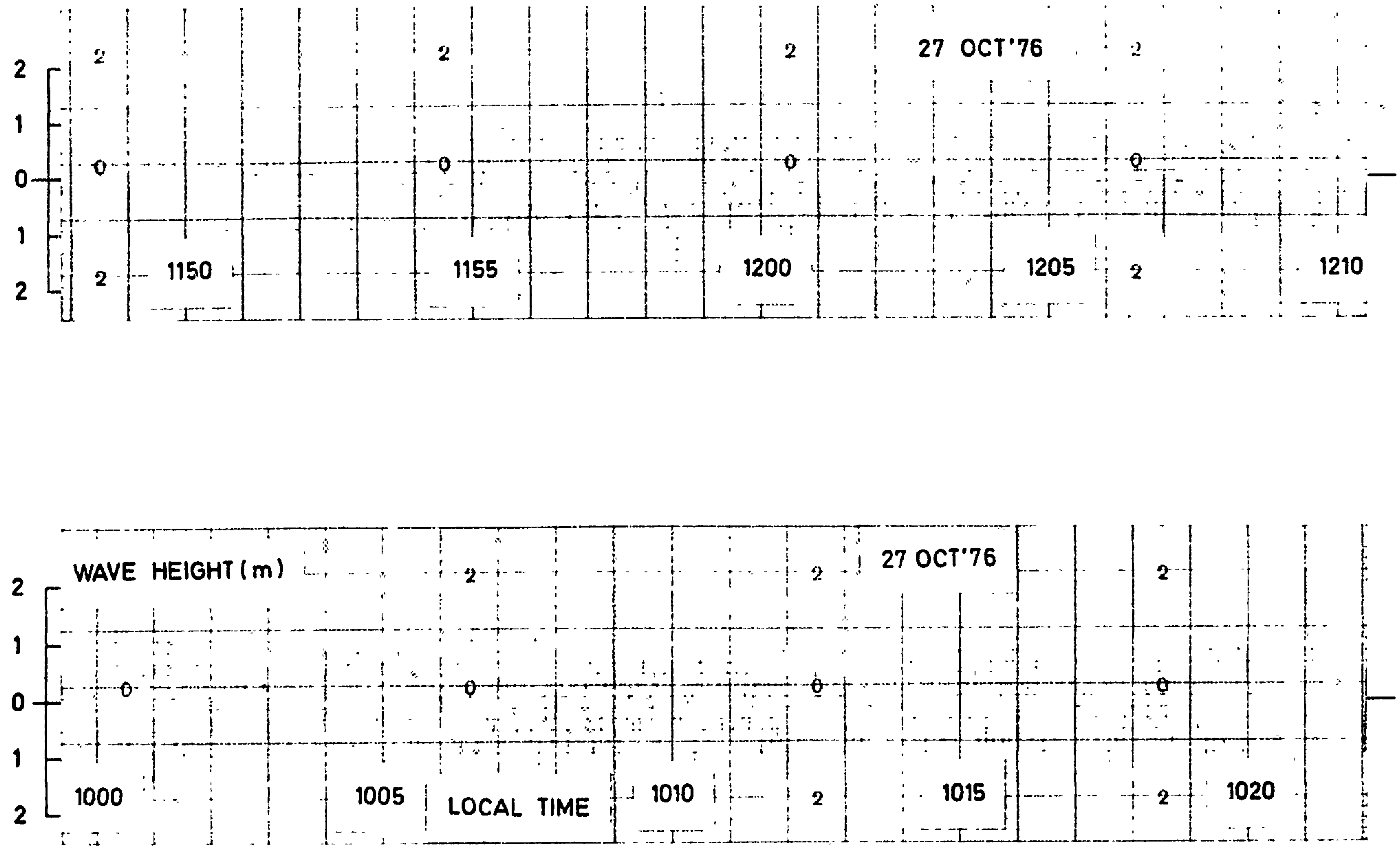


Fig.14. Wave height record 1001 27 OCT'76 and 1148 27 OCT'76.

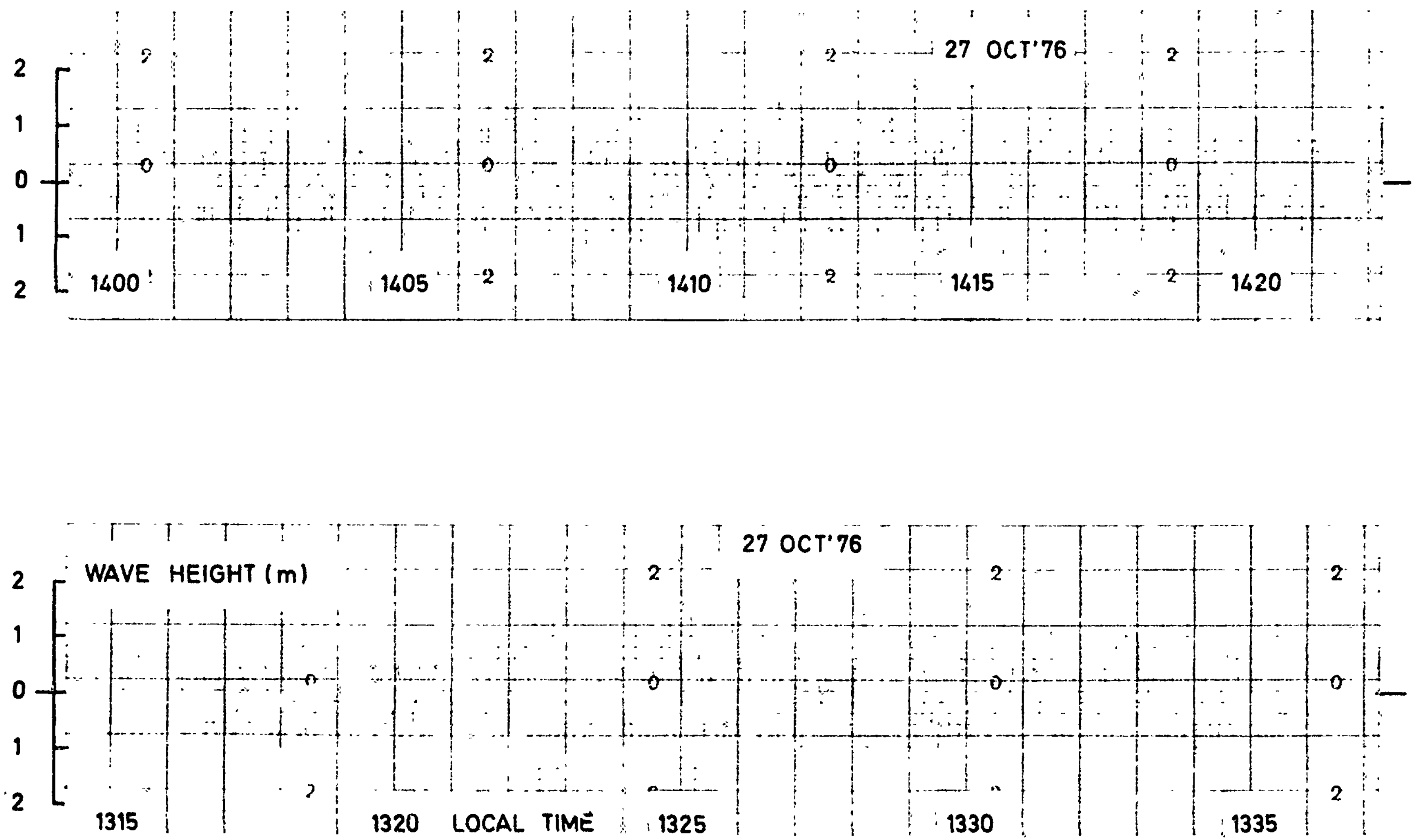


Fig.15. Wave height record 1315 27 OCT'76 and 1400 27 OCT'76.

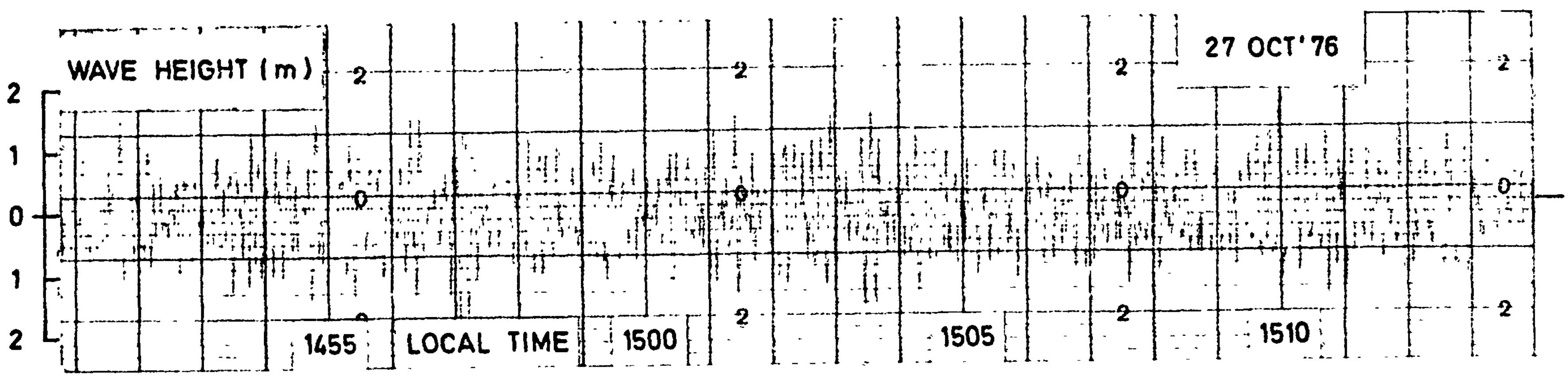


Fig.16. Wave height record 1451 27 OCT'76.

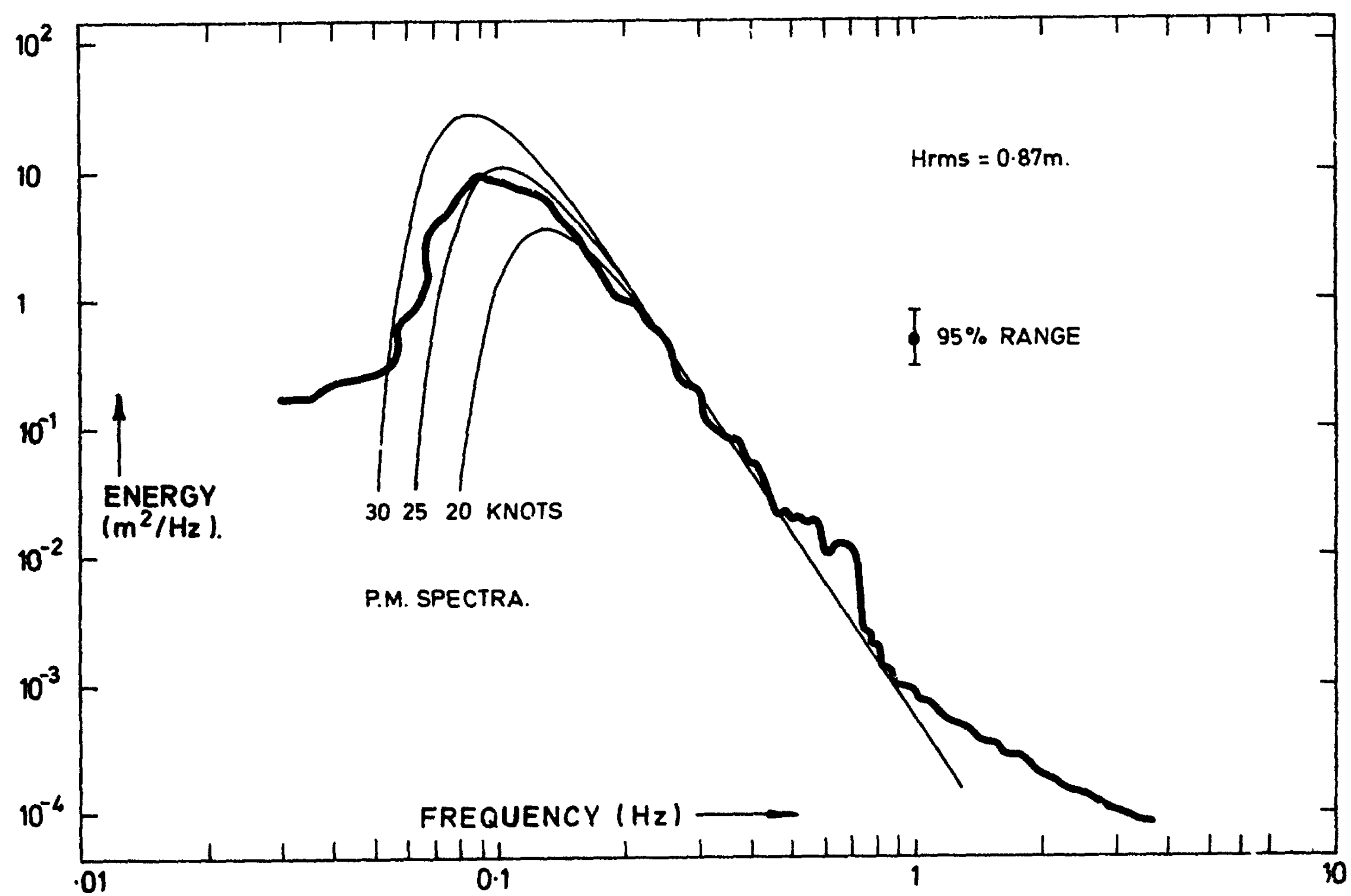


Fig.17. Wave spectrum 1144 25 Oct. 76.

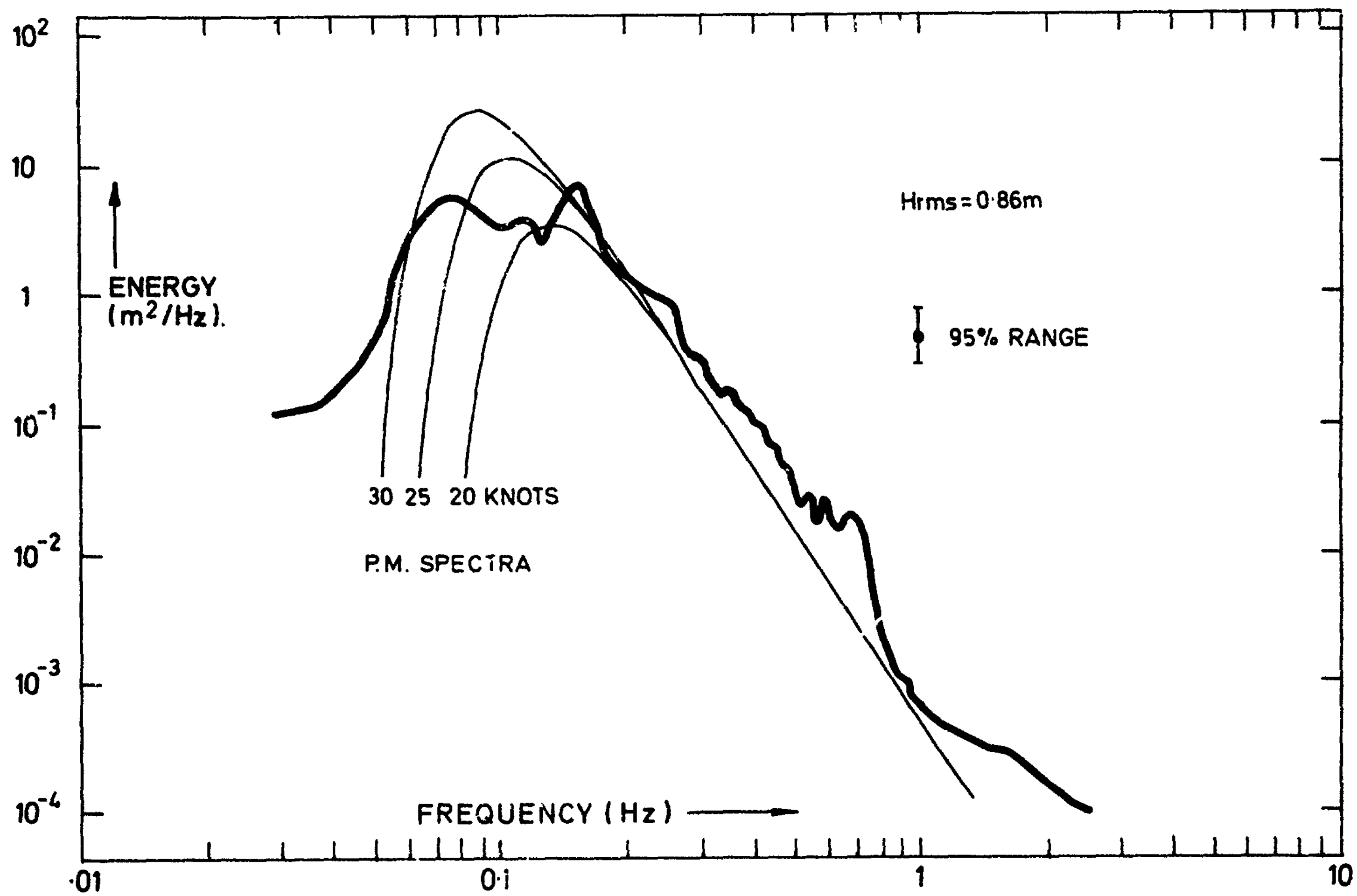


Fig.19. Wave spectrum 1040 26 Oct. 76.

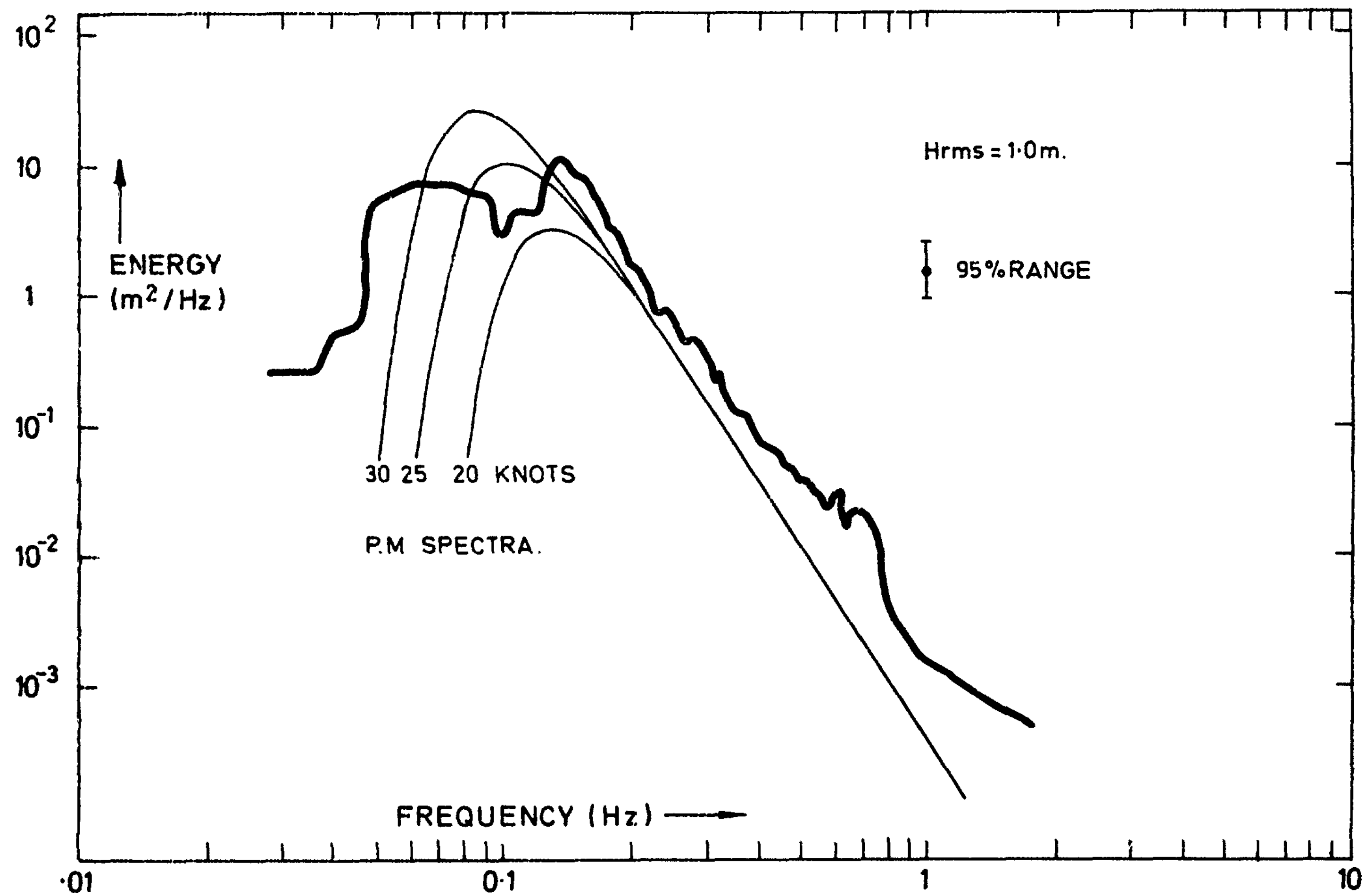


Fig. 18. Wave spectrum 0932 26 Oct. 76.

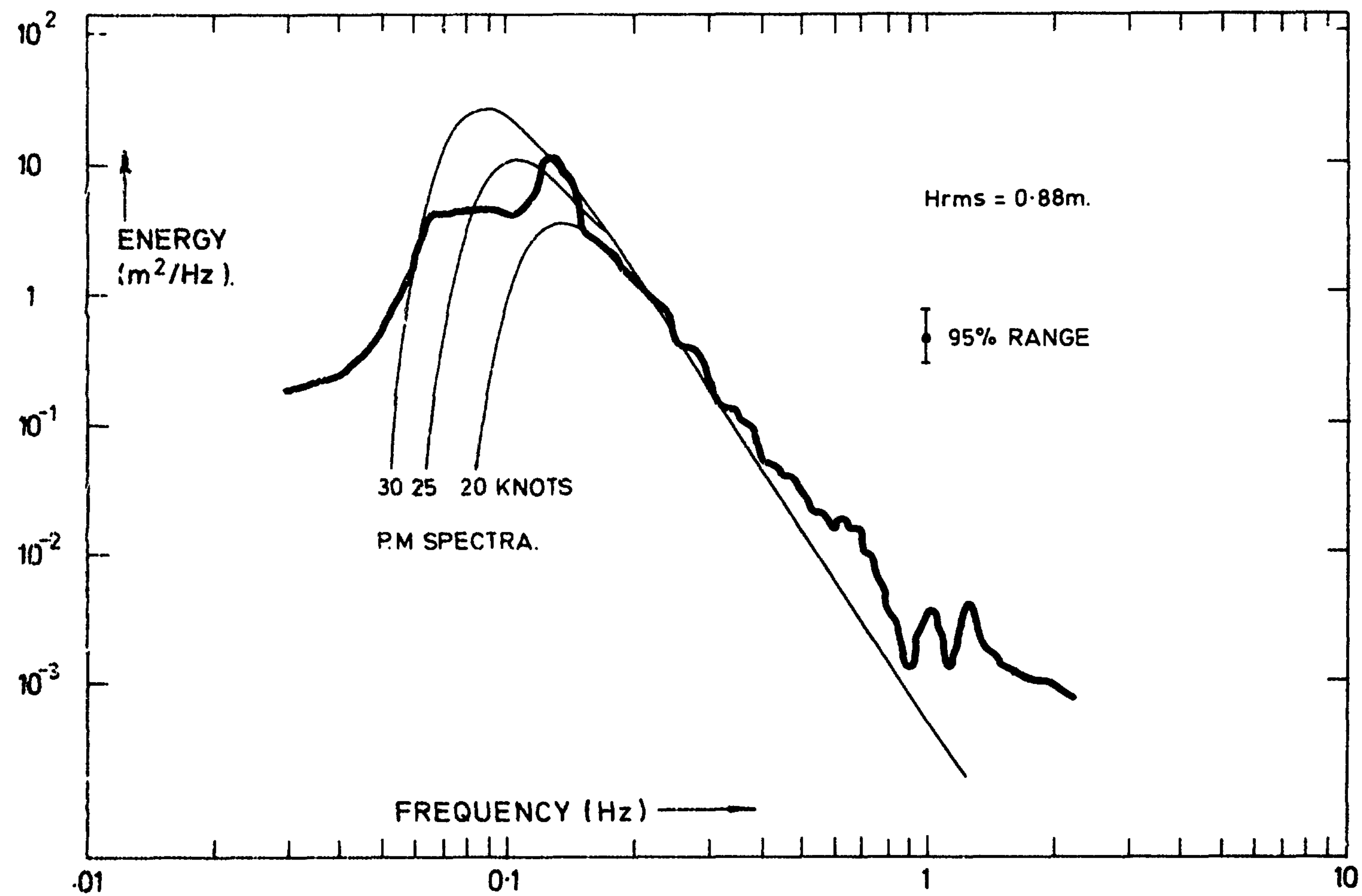


Fig.20. Wave spectrum 1614 . 26 Oct. 76.

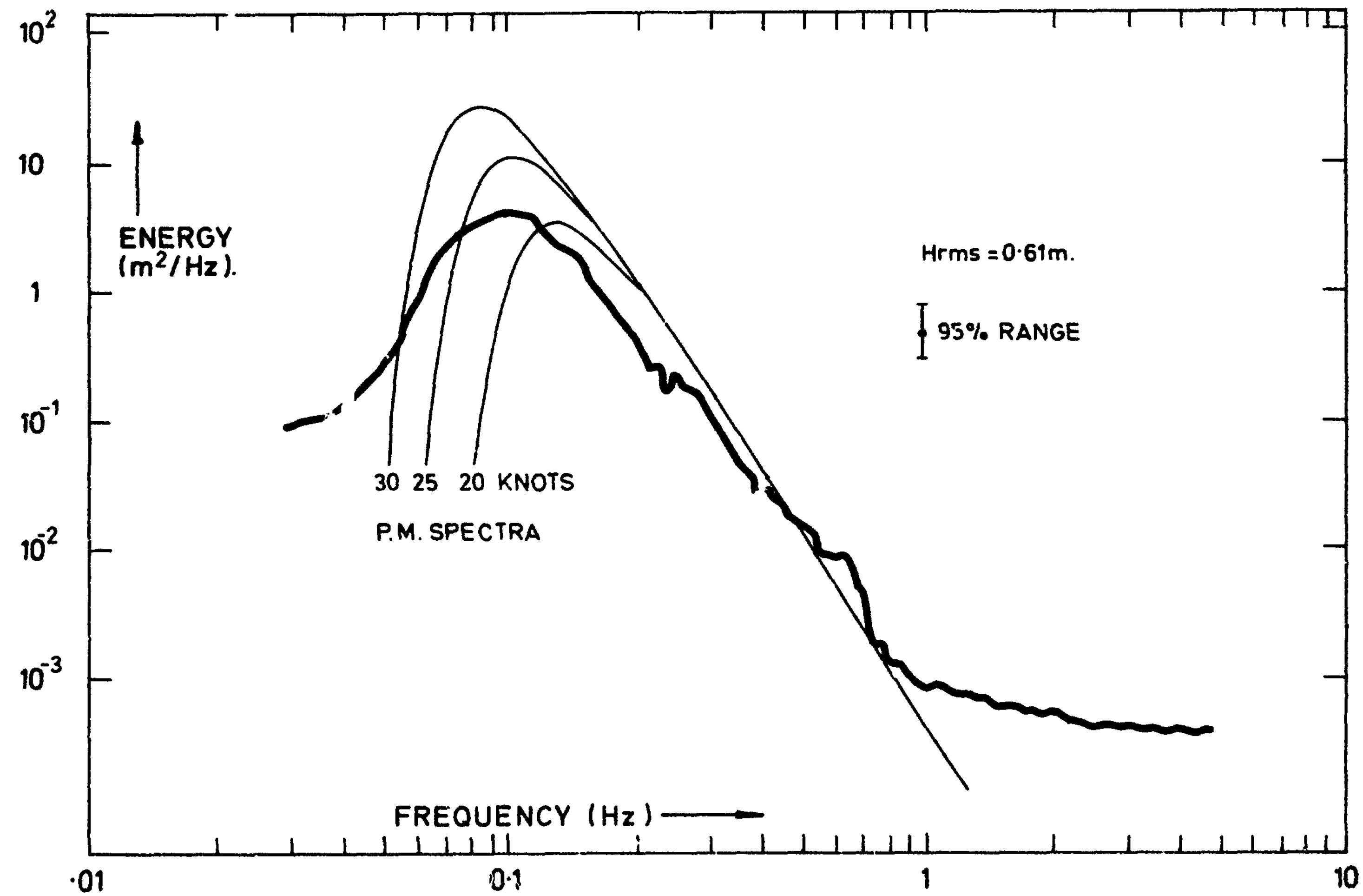


Fig.21. Wave spectrum 1001 27 Oct.76.

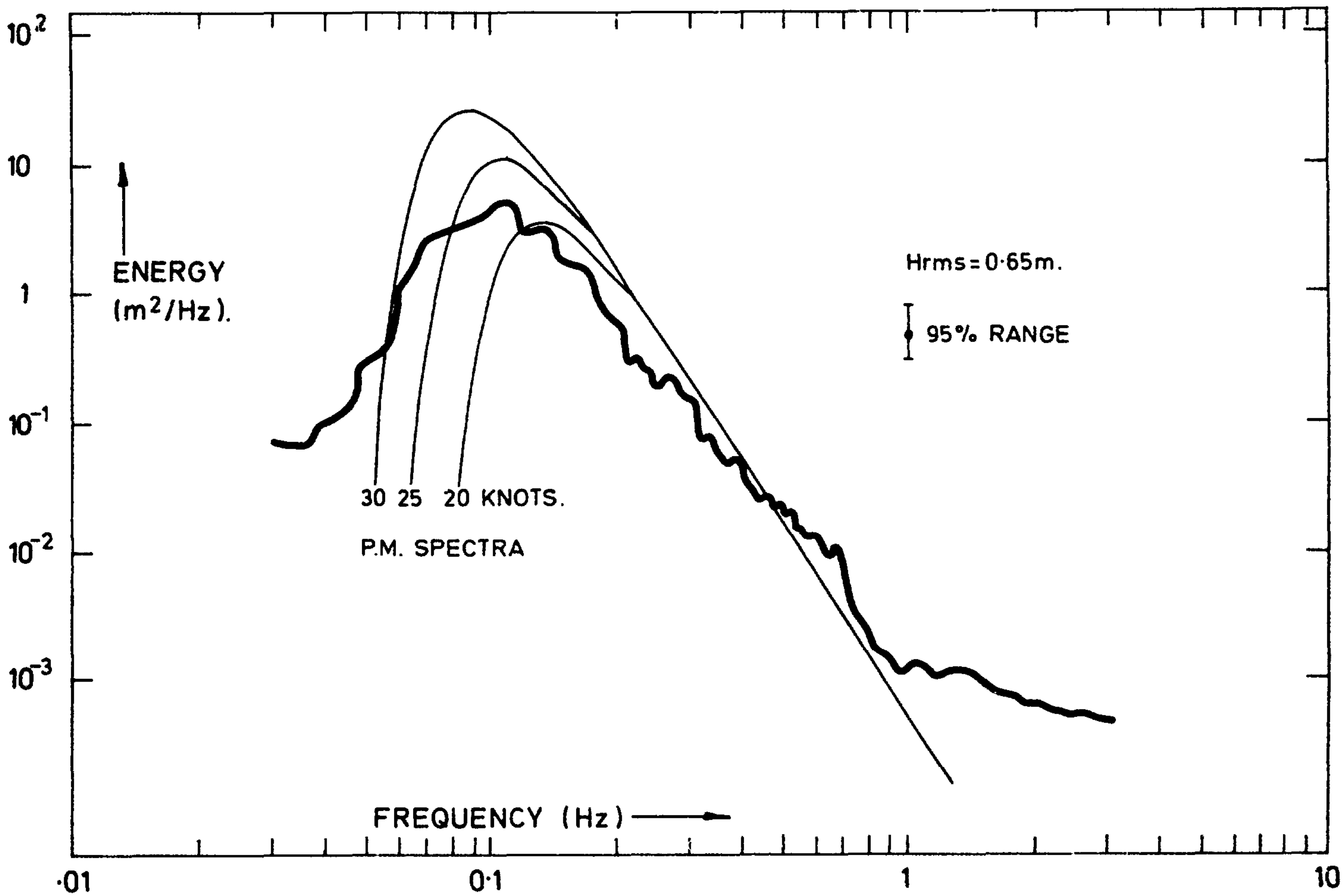


Fig. 22. Wave spectrum 1145 27 Oct. 76.

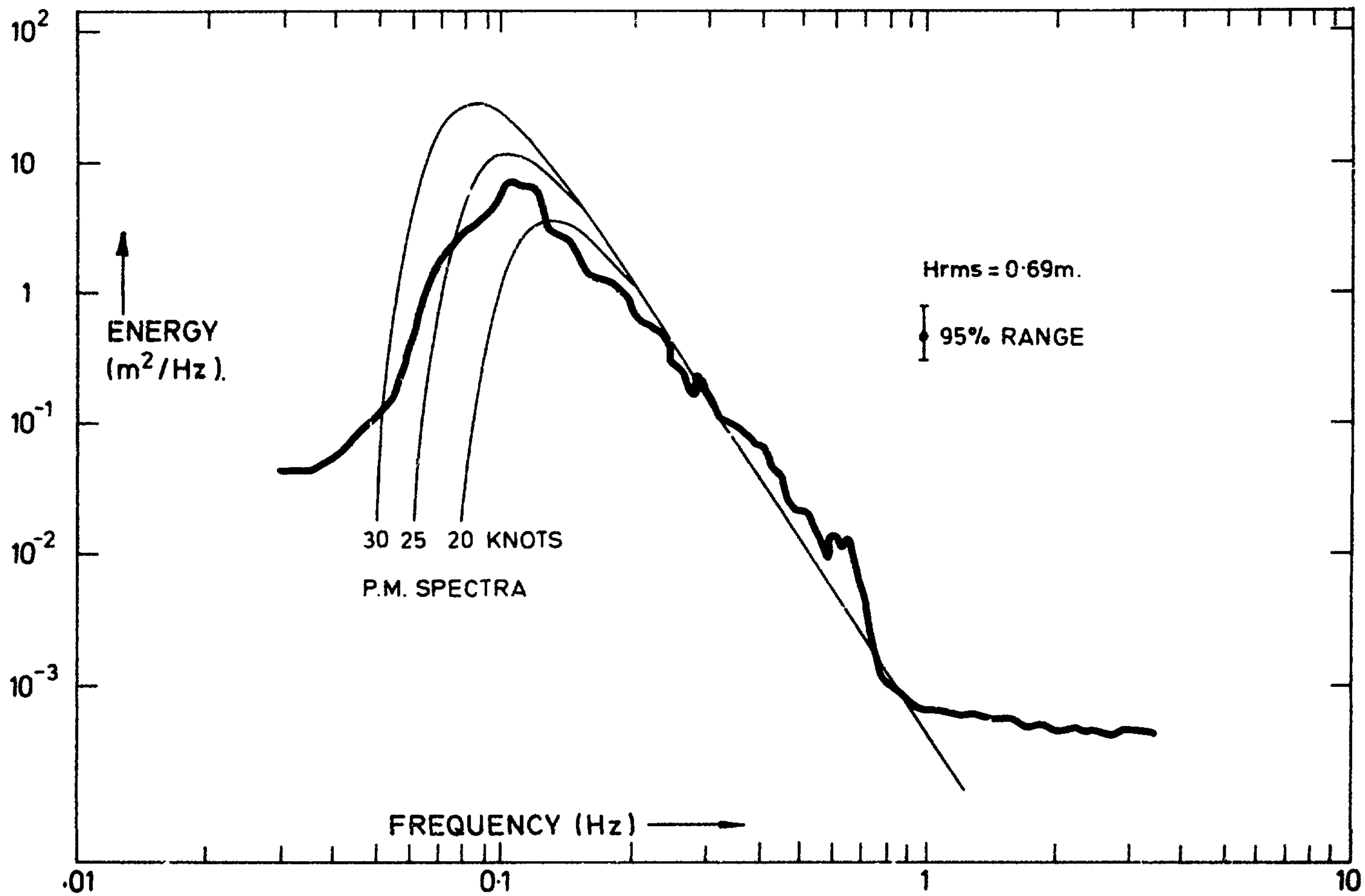


Fig. 23. Wave spectrum 1230 27 Oct. 76.

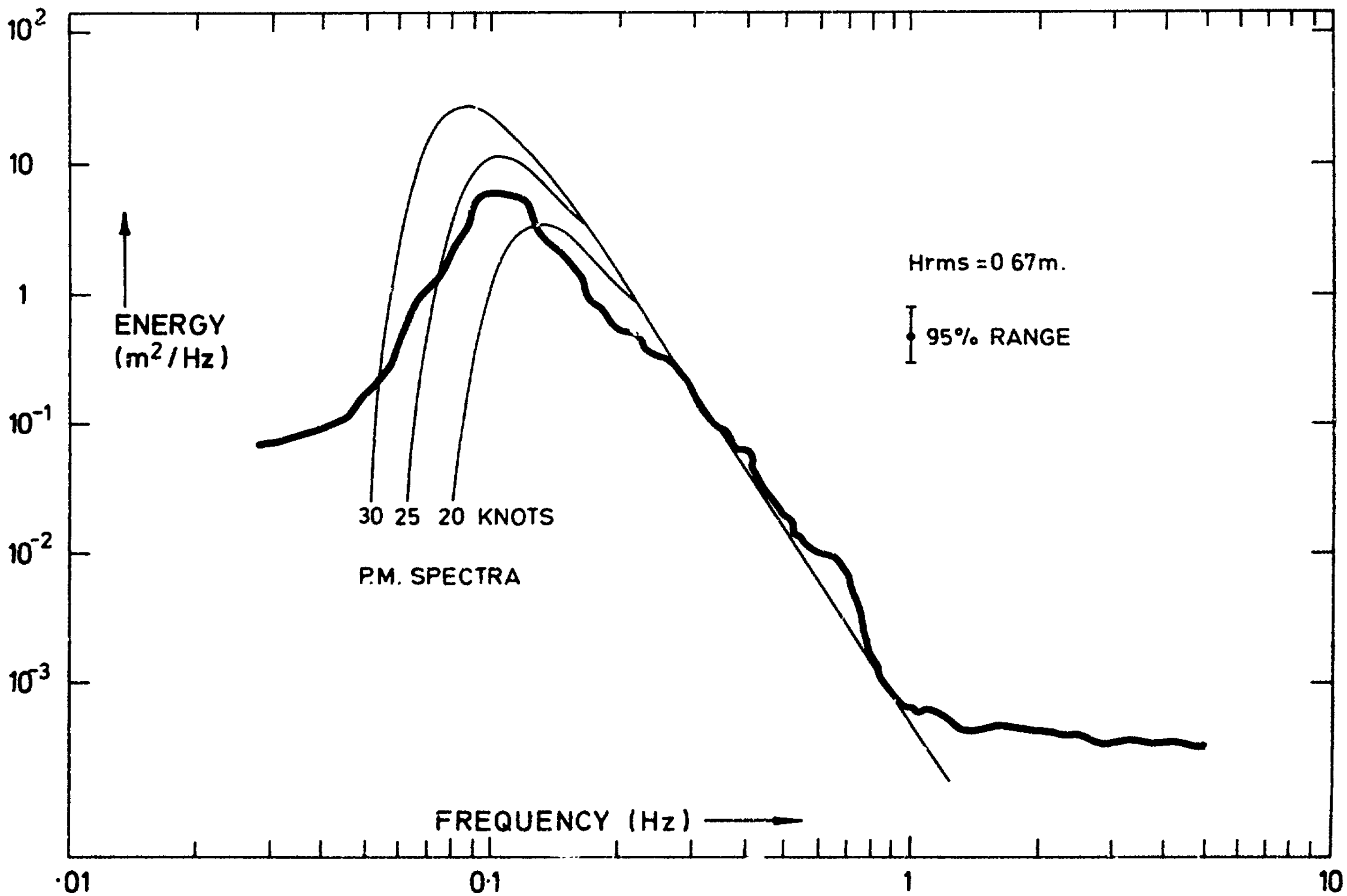


Fig. 24. Wave spectrum 1315 27 Oct. 76.

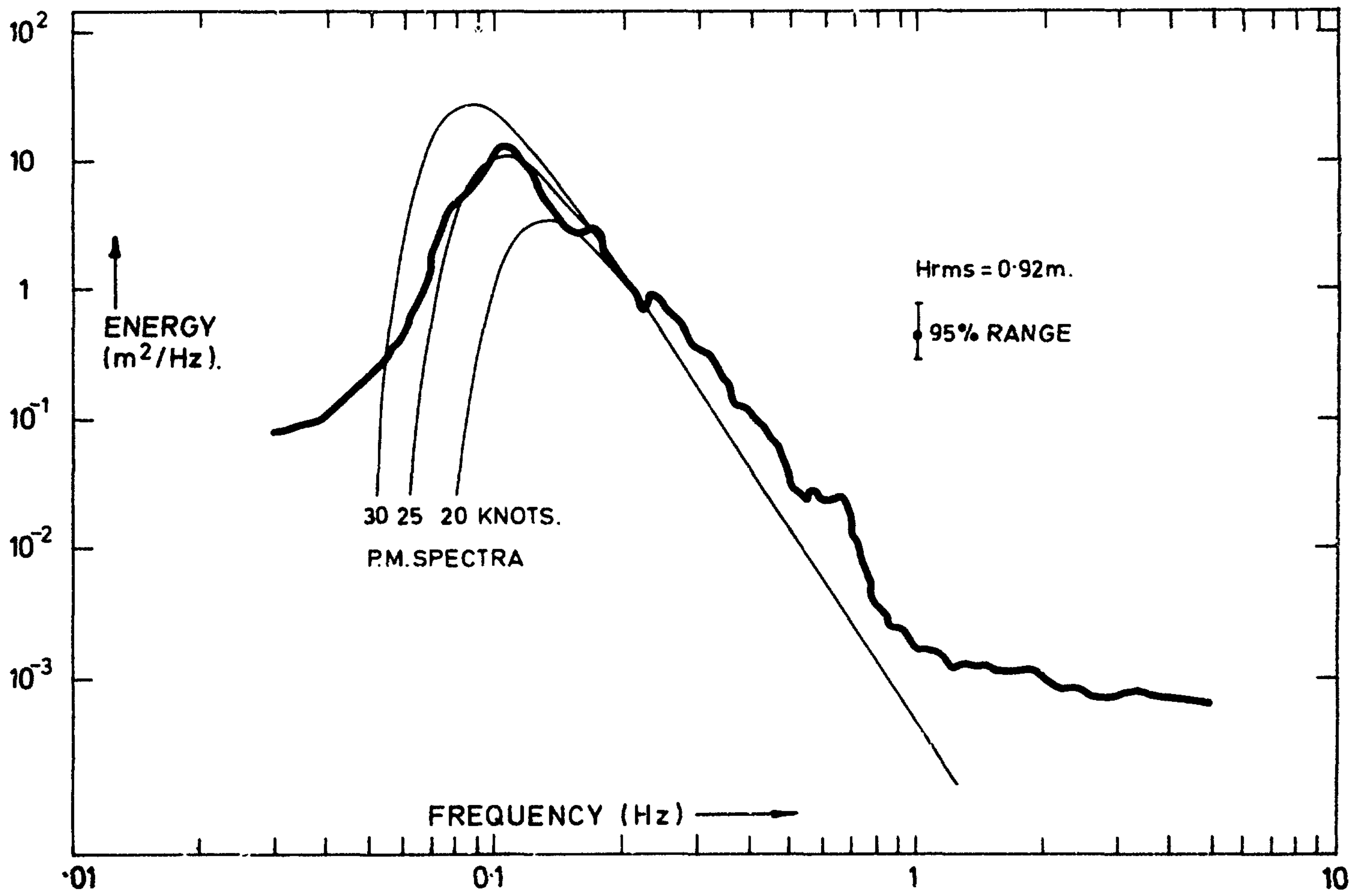


Fig.25 Wave spectrum 1400 27 Oct. 76.

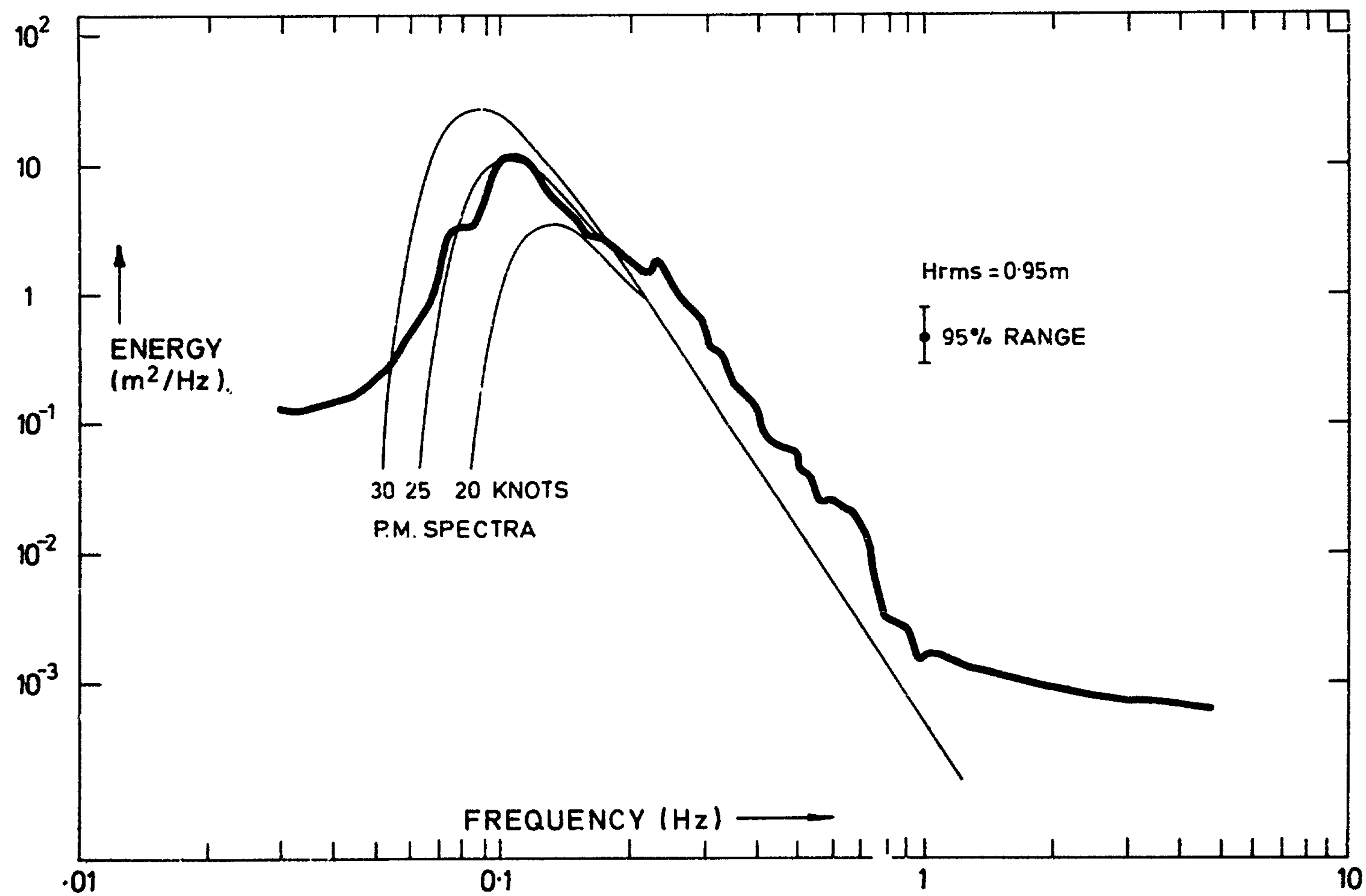


Fig.26. Wave spectrum 1451 27 Oct.76.

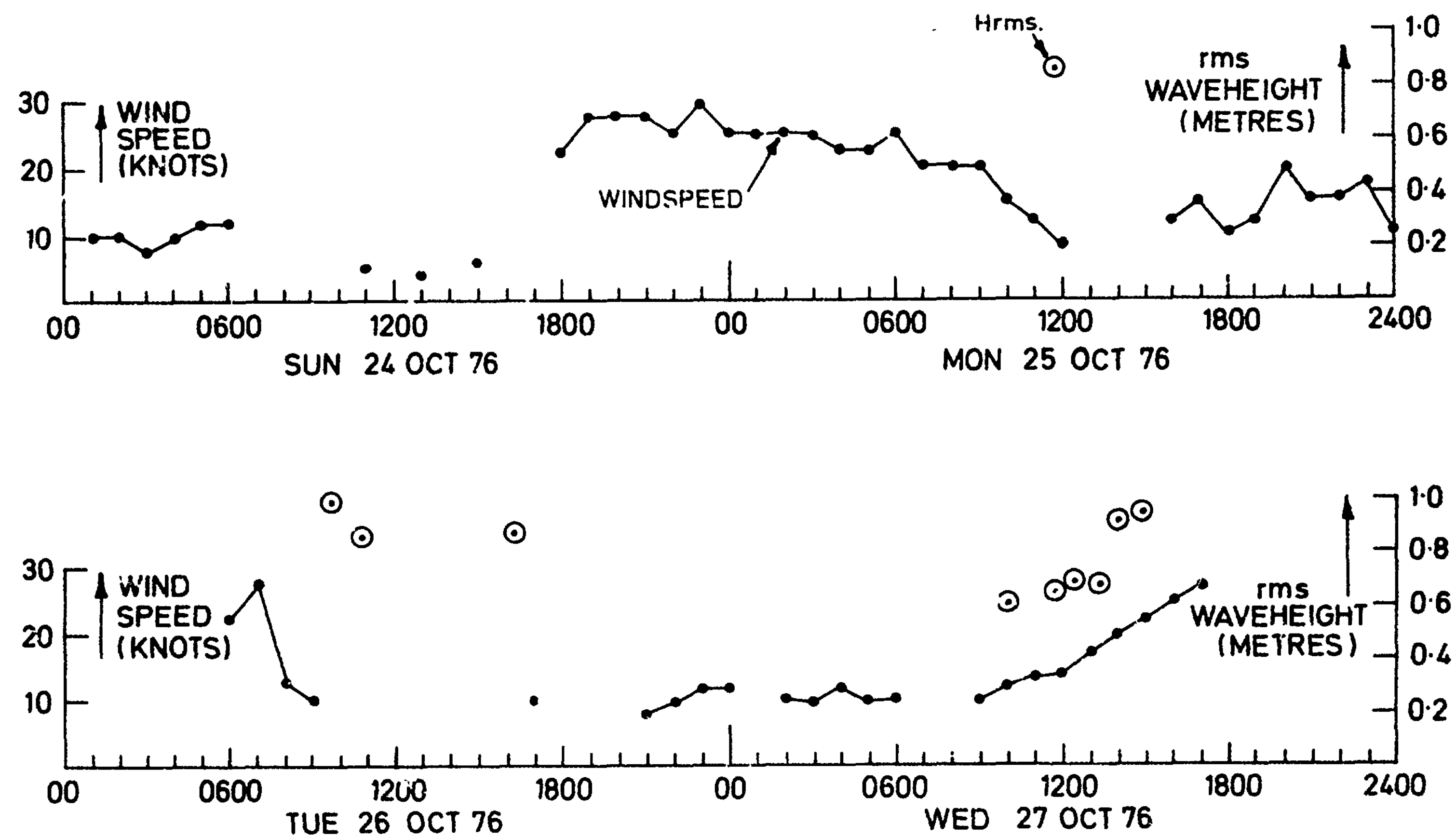


Fig. 27. Wind speed and root mean square waveheight 24 to 27 OCT. 76.

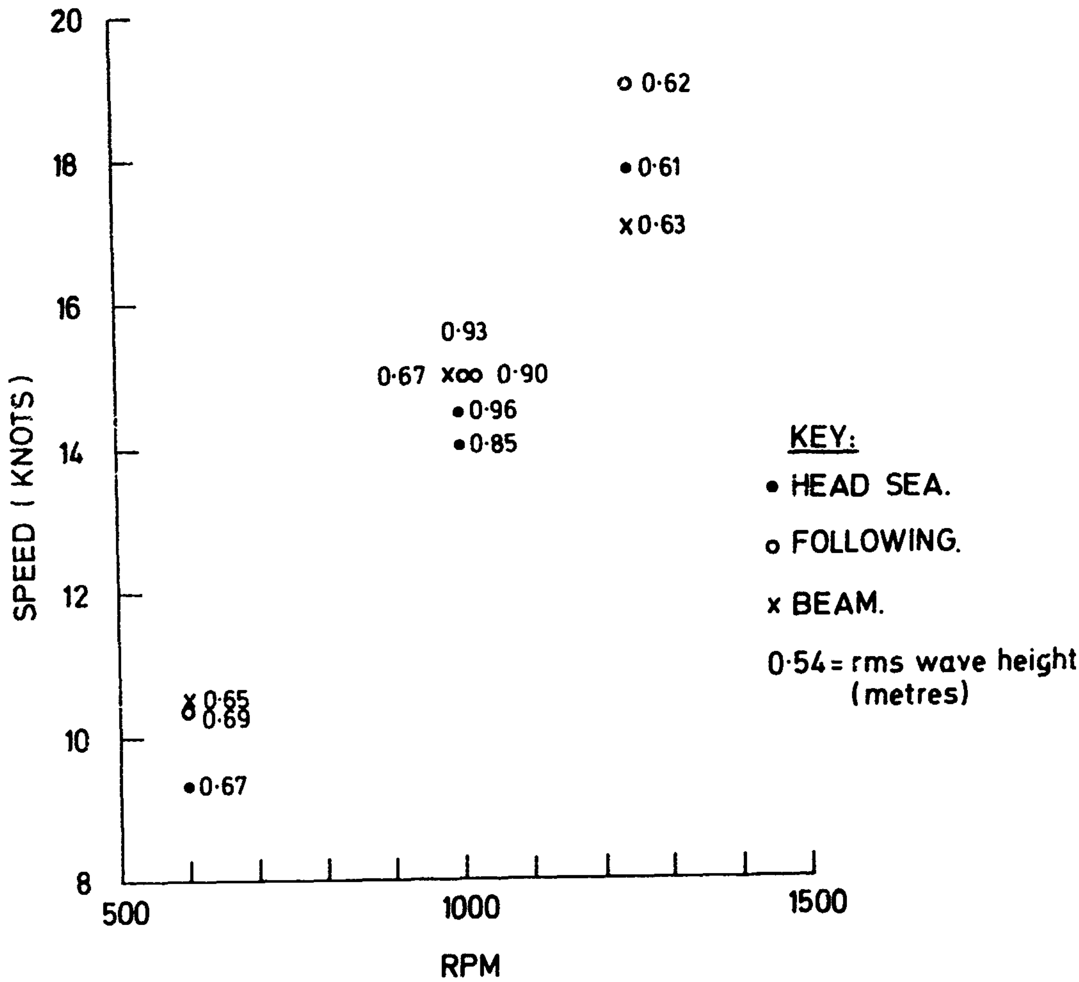


Fig. 28. Speed of Patrol Boat vs engine rpm for different sea aspects.

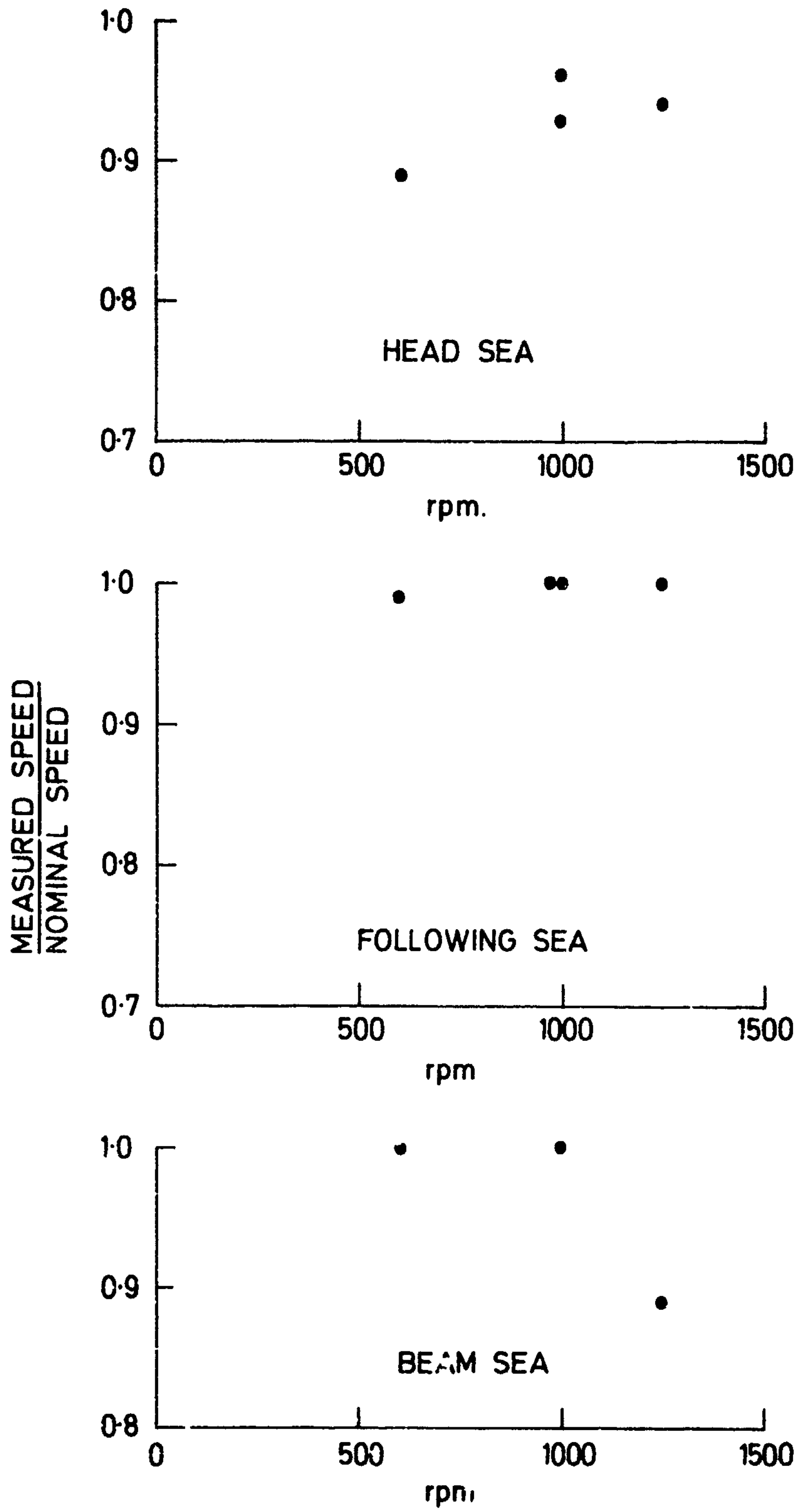


Fig. 29. Comparison between measured speed and nominal speed.

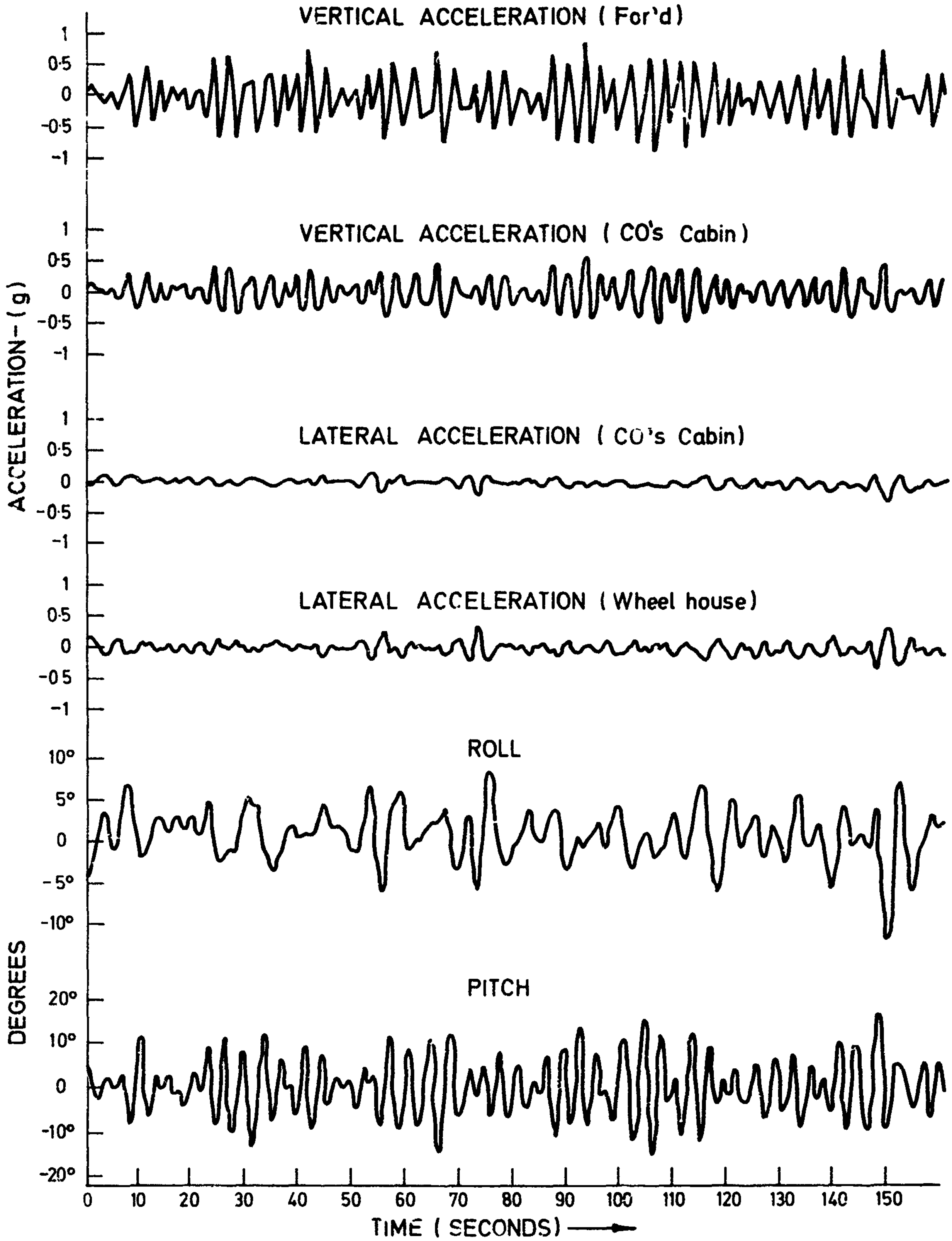


Fig.30. Patrol Boat motion. Run Nº1 26 OCT'76
15 knots. Head sea.

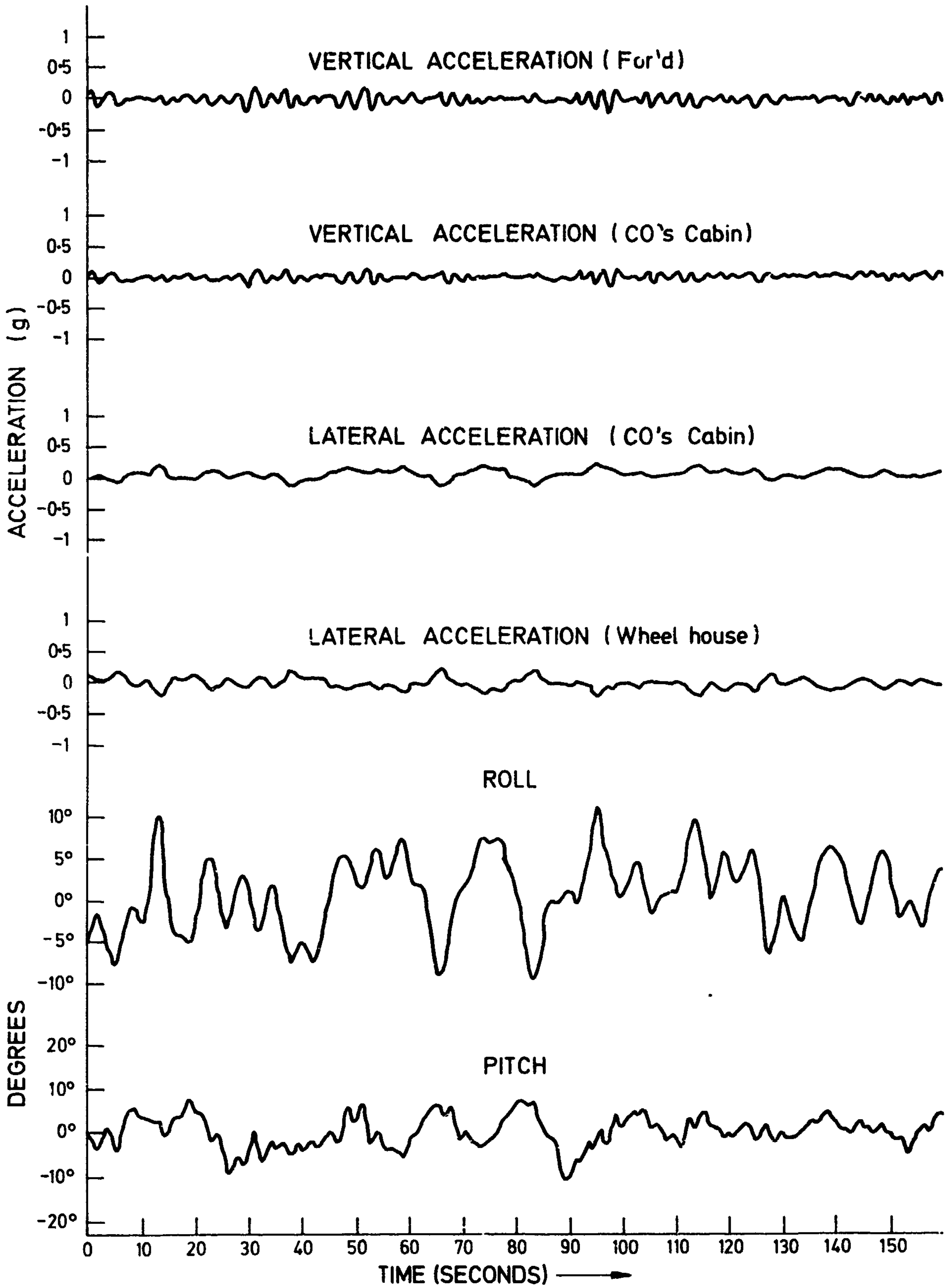


Fig.31. Patrol Boat motion. Run Nº2 26 OCT'76
15 knots. Following sea.

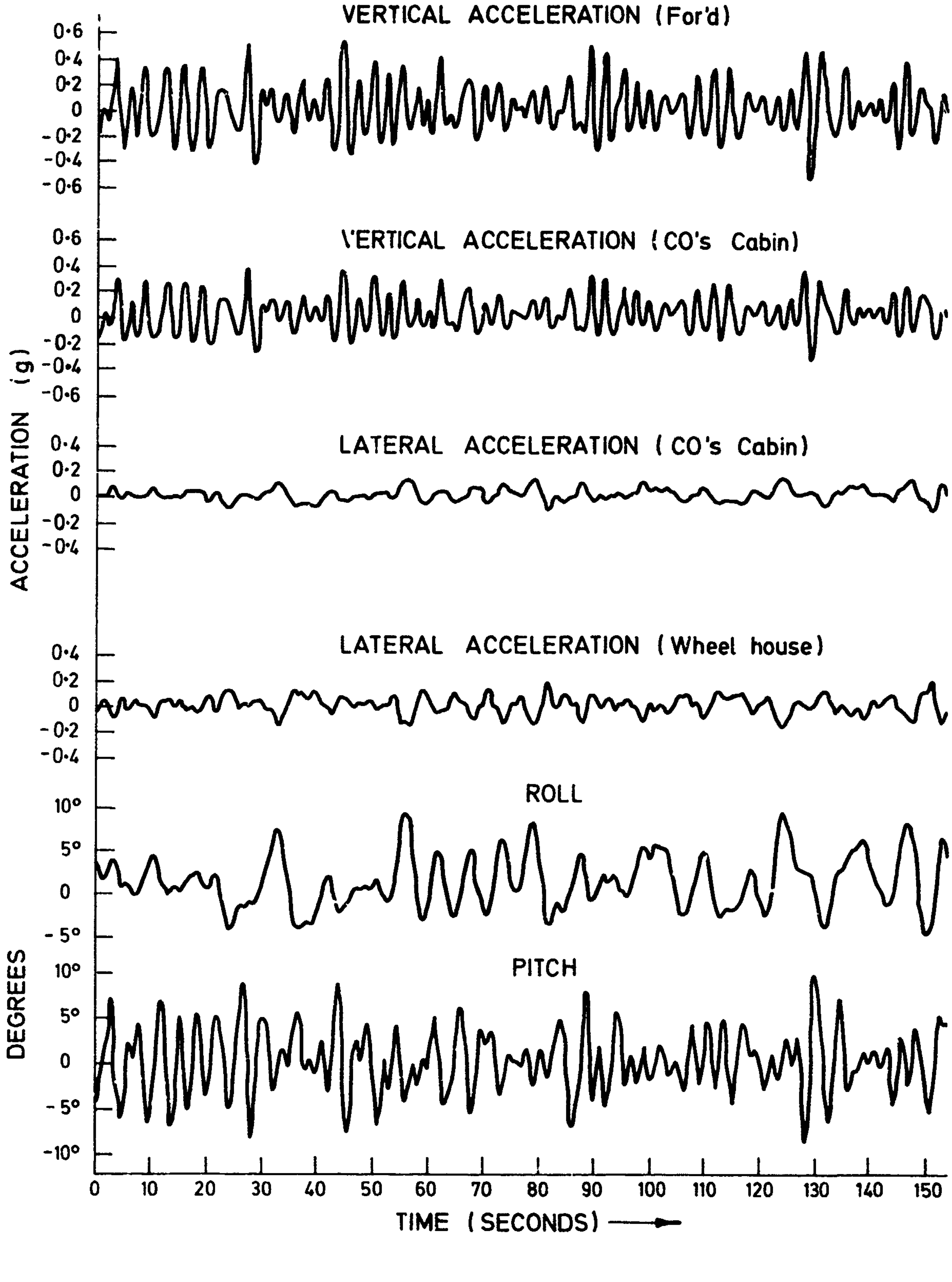


Fig.32. Patrol Boat motion. Run №1 27 OCT '76
19 knots. Head sea.

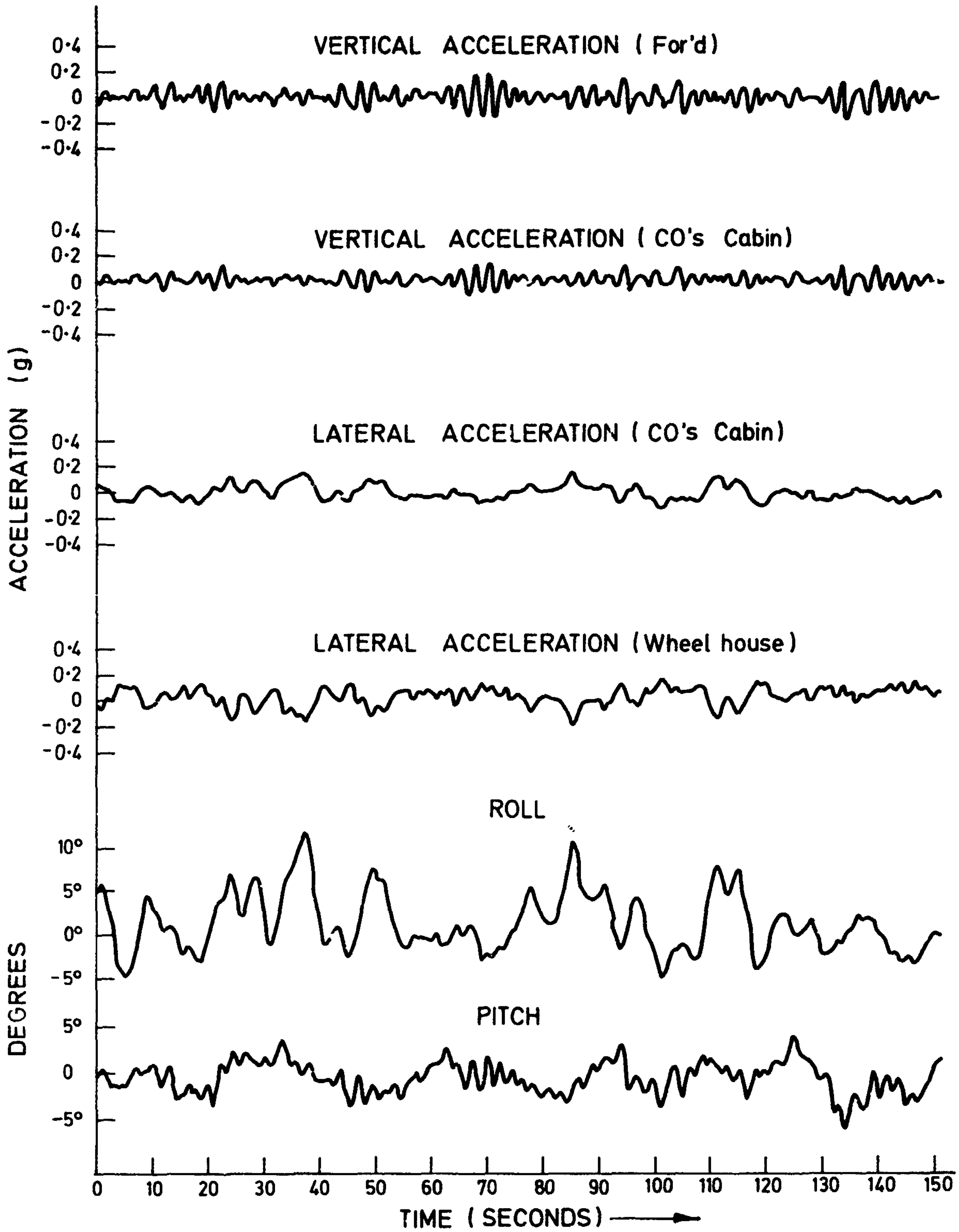


Fig.33. Patrol Boat motion. Run Nº 2 27 OCT'76
19 knots. Following sea.

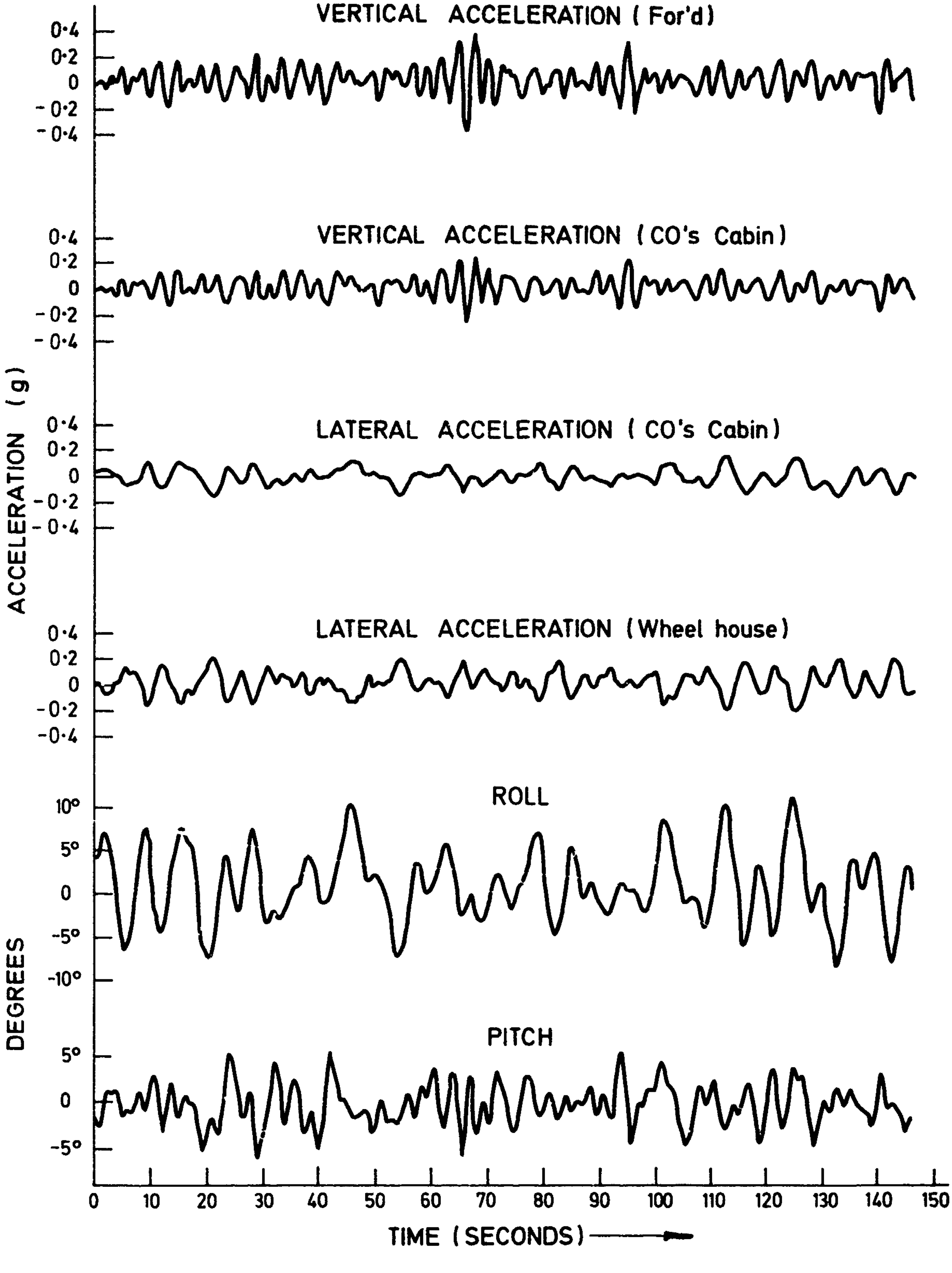


Fig.34. Patrol Boat motion. Run Nº3 27 OCT'76
19 knots. Beam sea.

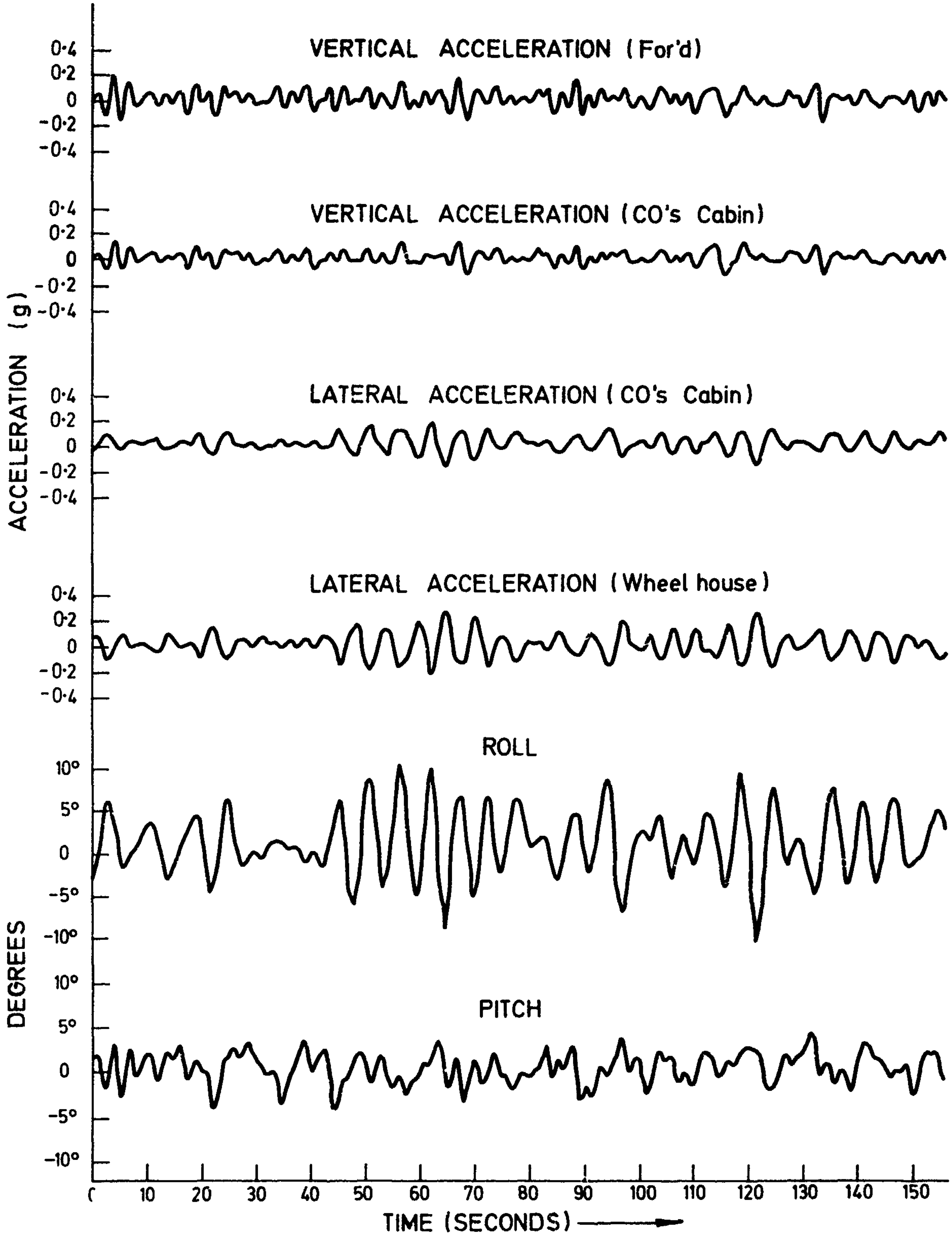


Fig.35. Patrol Boat motion. Run No.4 27 OCT '76
10 knots. Beam sea.

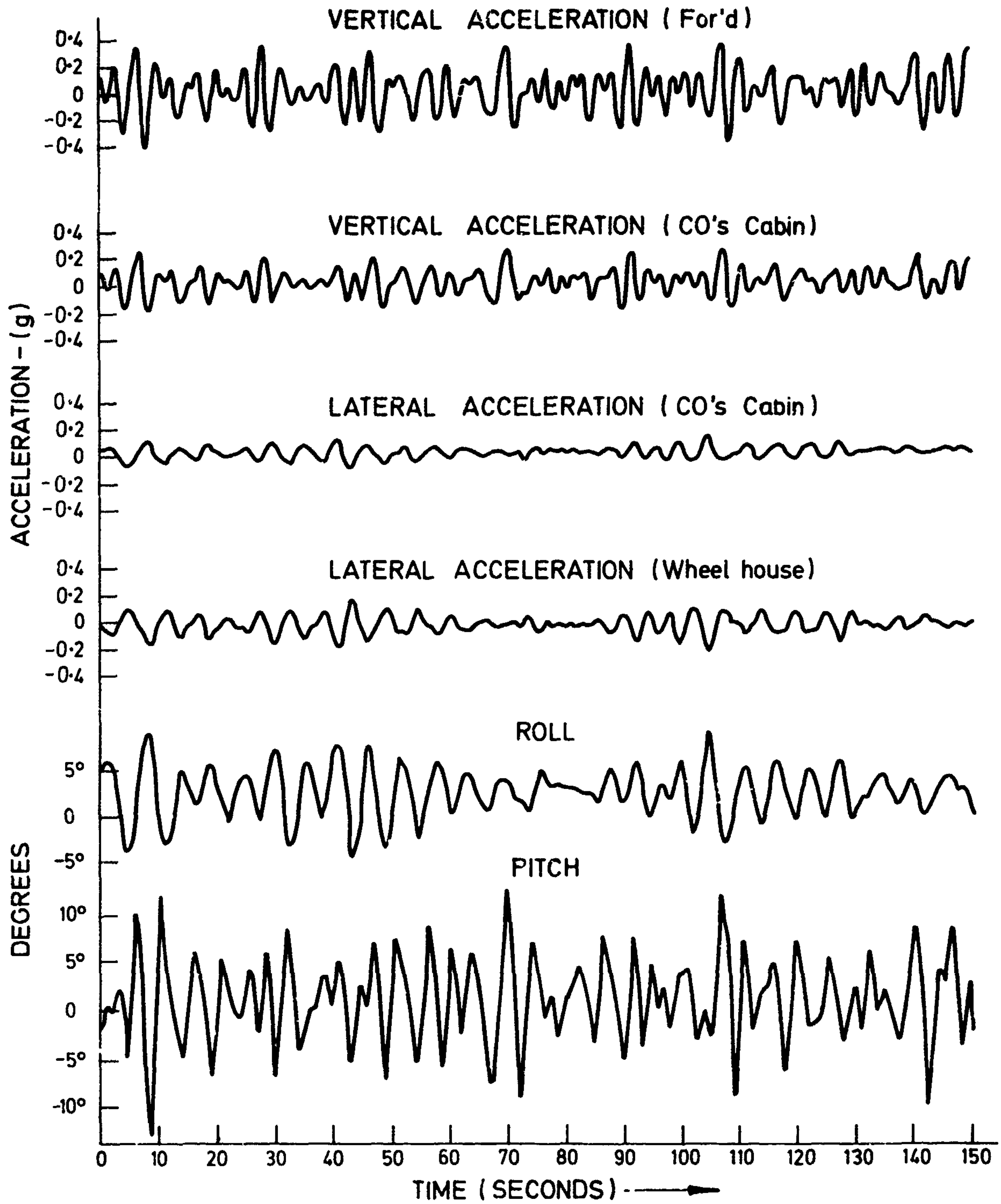


Fig.36. Patrol Boat motion. Run Nº5 27 OCT'76
10 knots. Head sea.

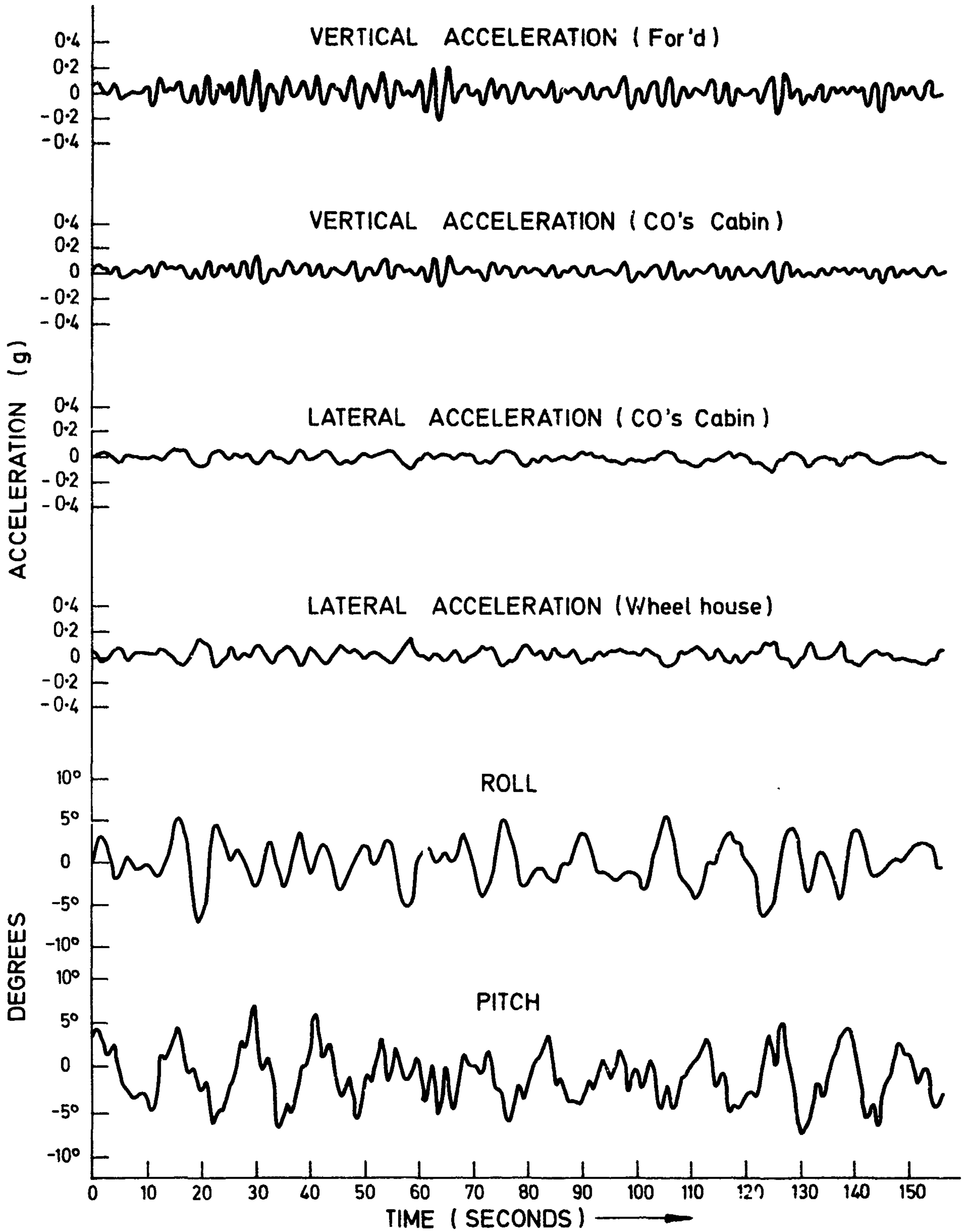


Fig. 37. Patrol Boat motion. Run Nº6 27 OCT'76
10 knots. Following sea.

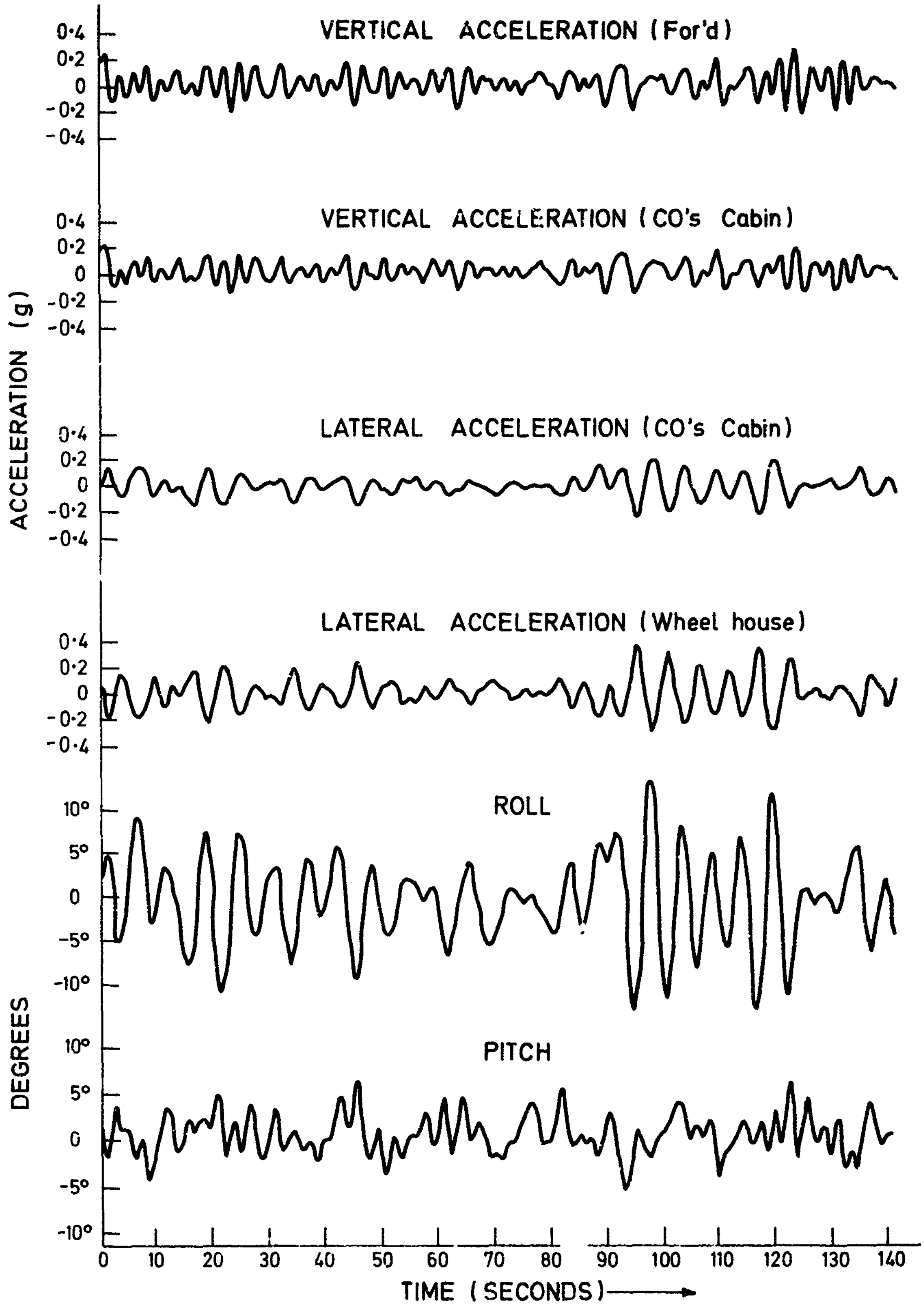


Fig.38. Patrol Boat motion. Run N°7 27 OCT'76
15 knots. Beam sea.

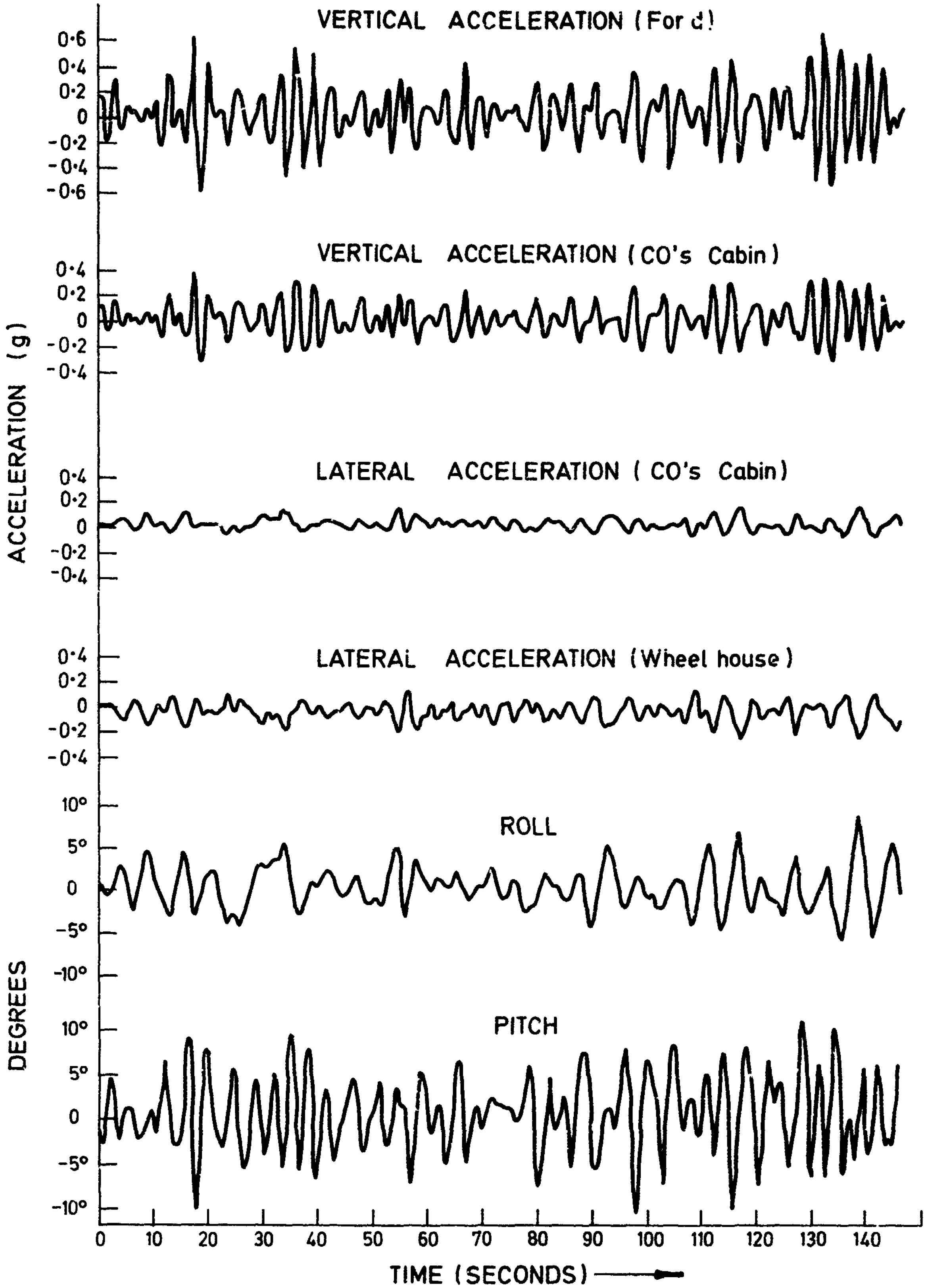


Fig.39. Patrol Boat motion. Run Nº 8 27 OCT '76
15 knots. Head sea.

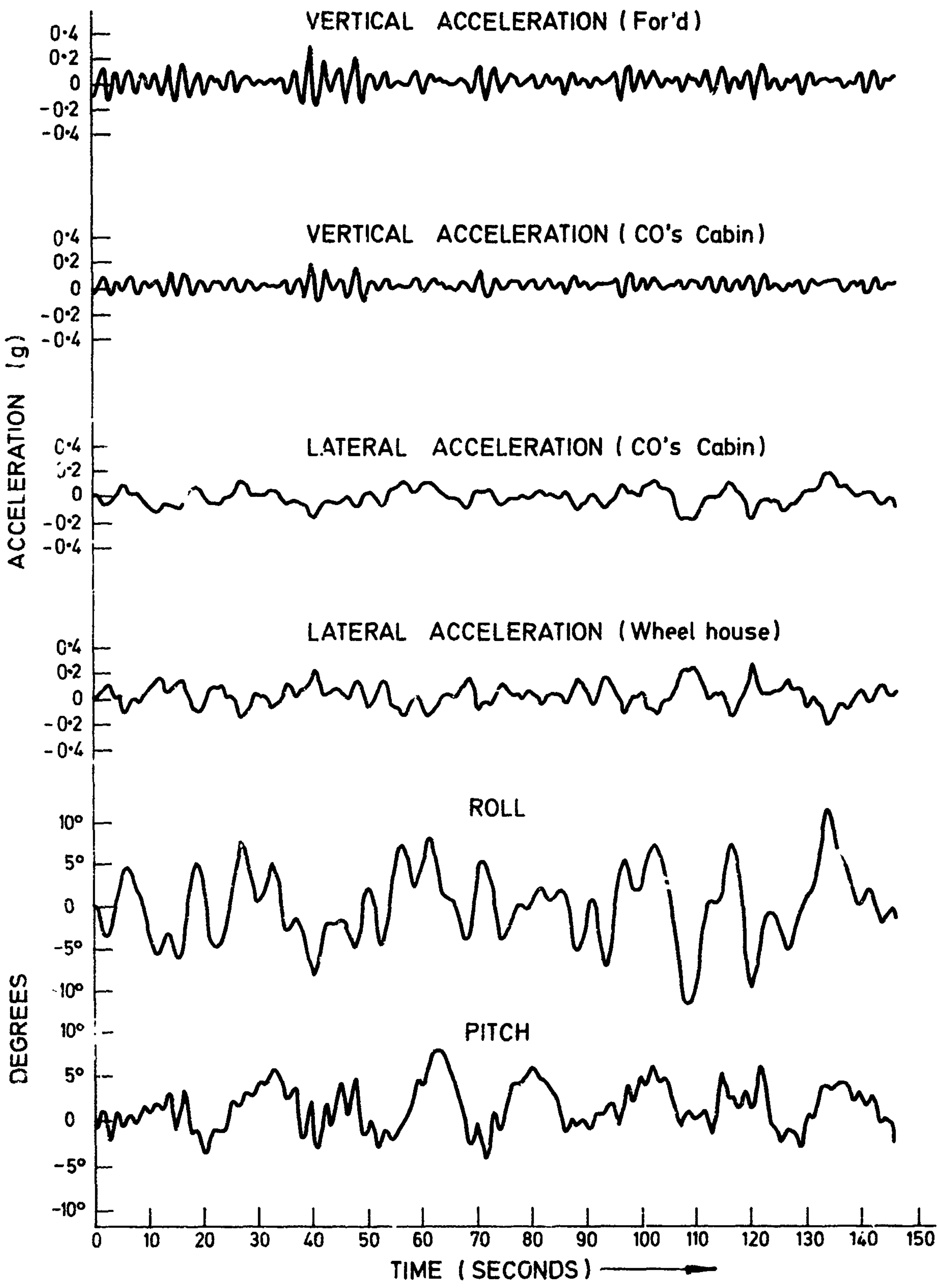


Fig. 40. Patrol Boat motion. Run Nº 9 27 OCT '76
15 knots. Following sea.

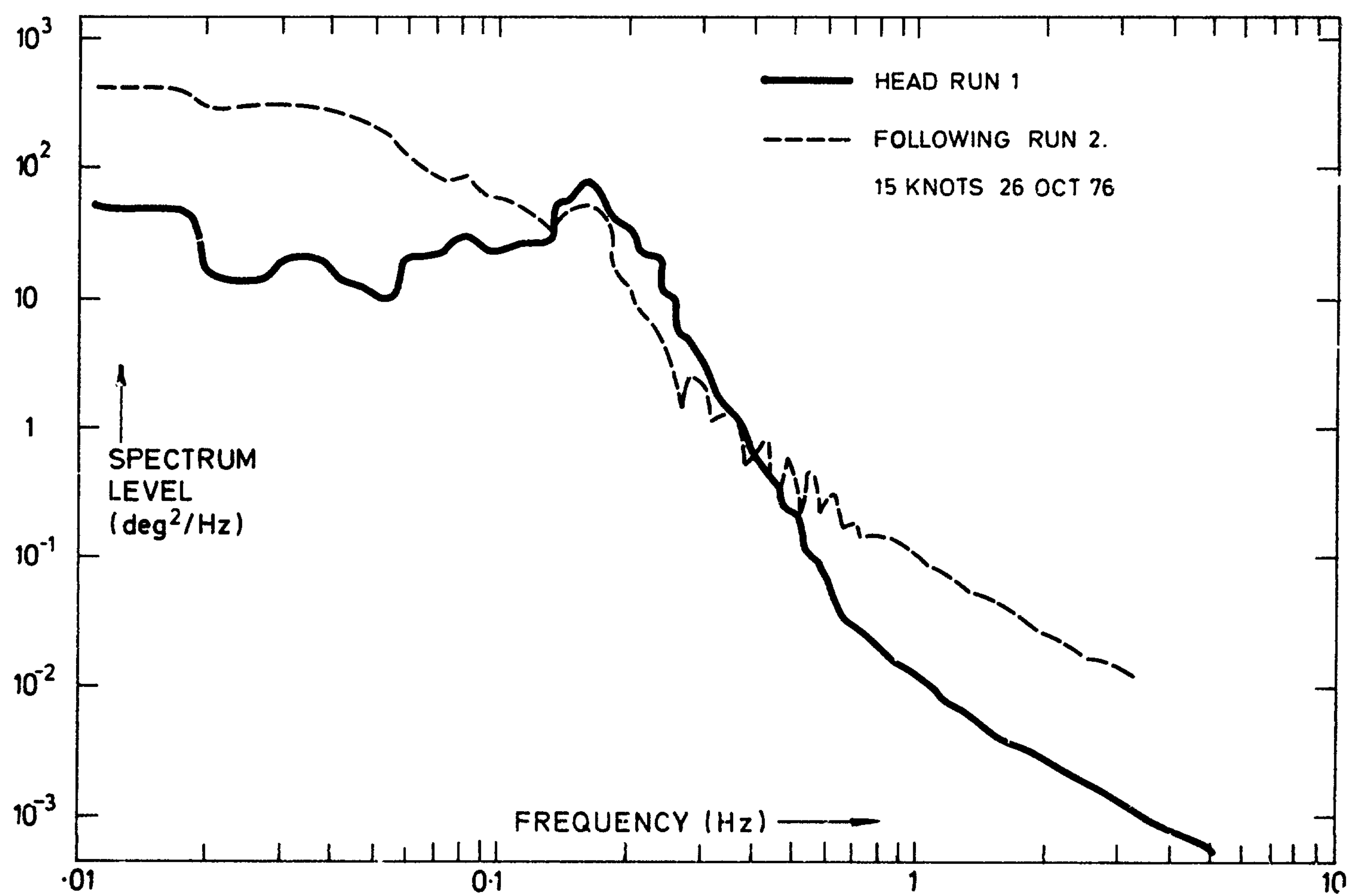


Fig. 41. Roil spectra at 15 knots. Runs 1&2 26 Oct. 76.

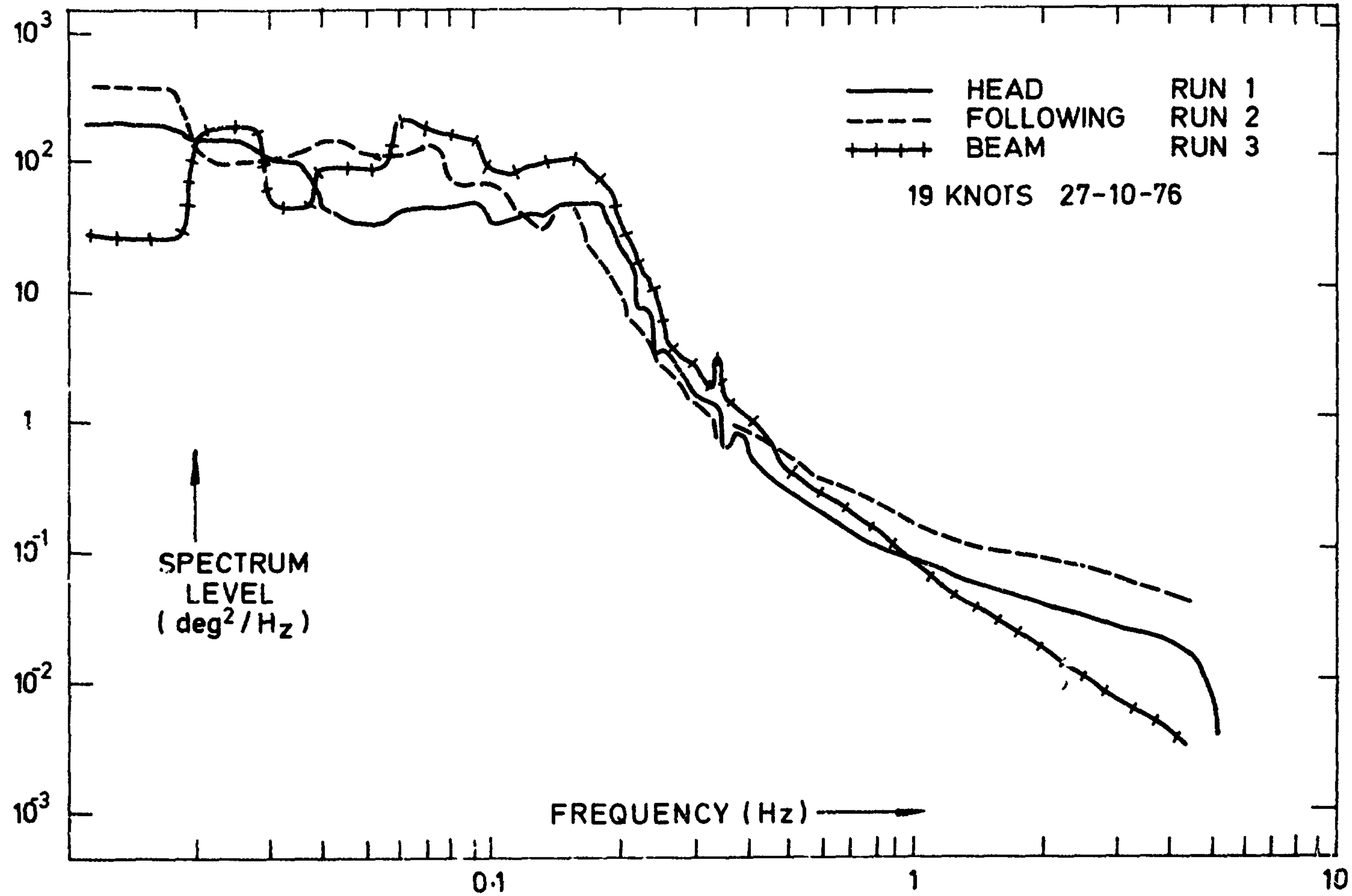


Fig.42. Roll spectra at 19 knots. Runs 1,2,3 27 Oct. 76

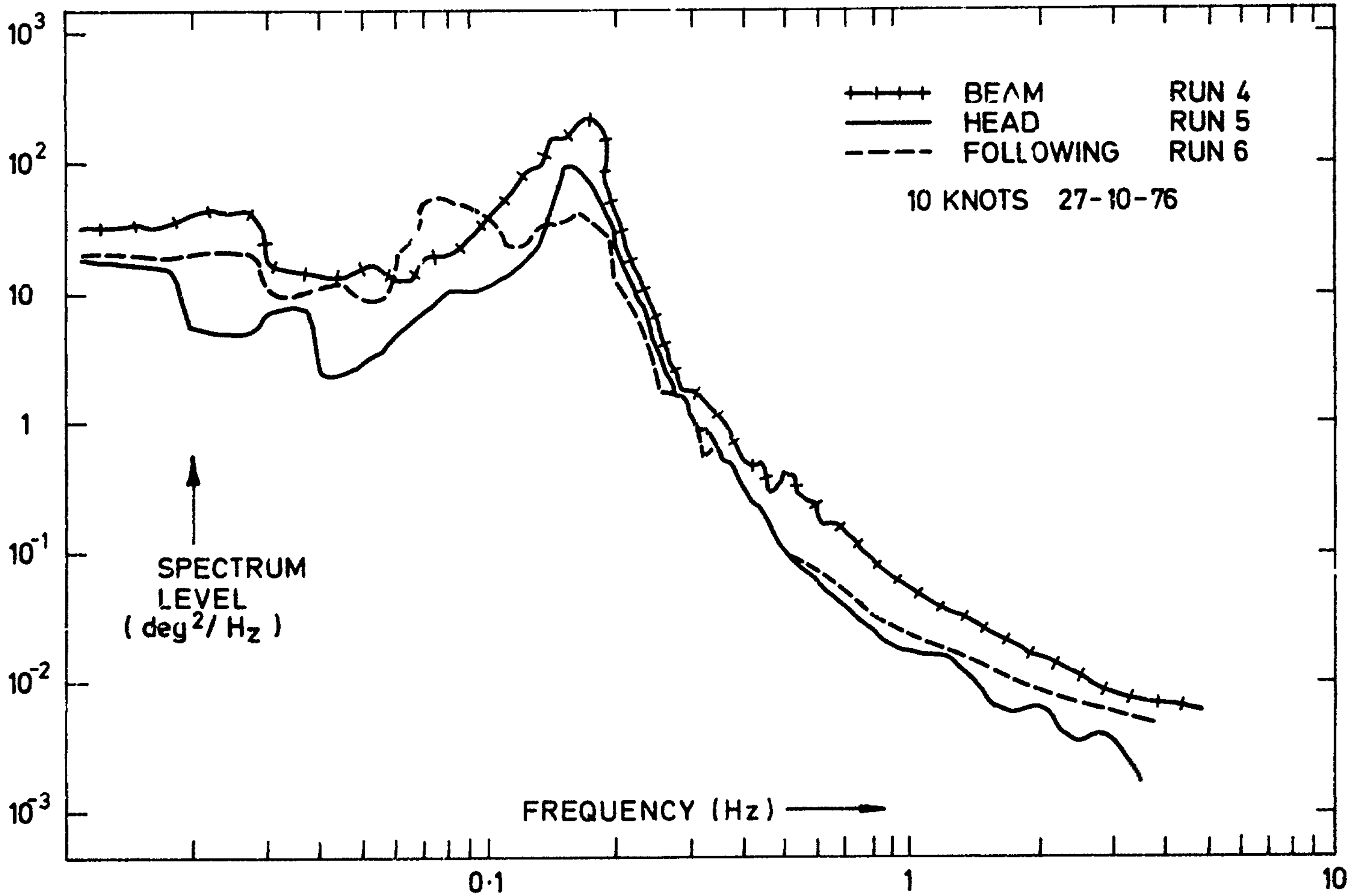


Fig.43. Roll spectra at 10 knots. Runs 4,5,6 27 Oct. 76.

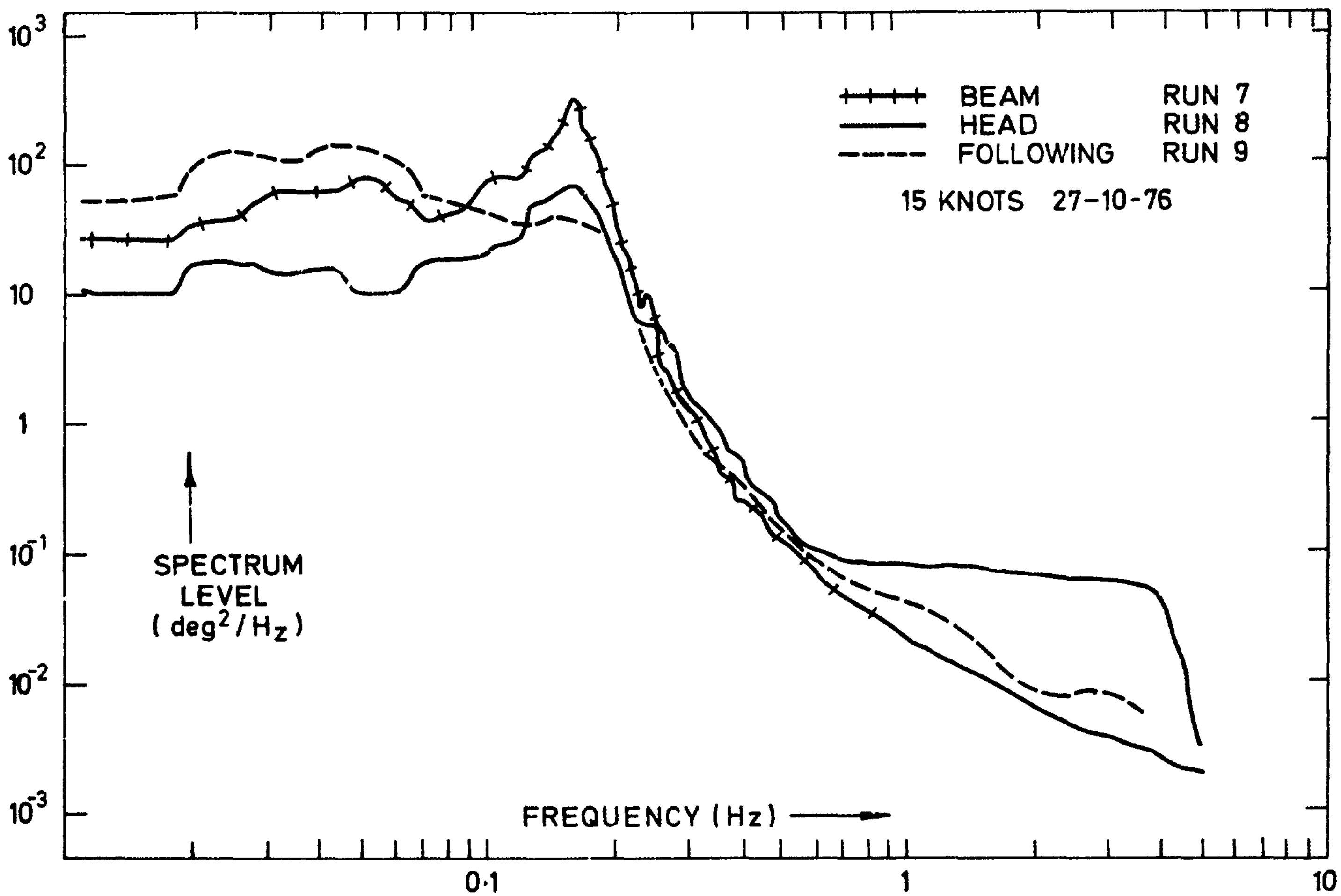


Fig.44. Roll spectra at 15 knots. Runs 7,8,9 27 Oct. 76.

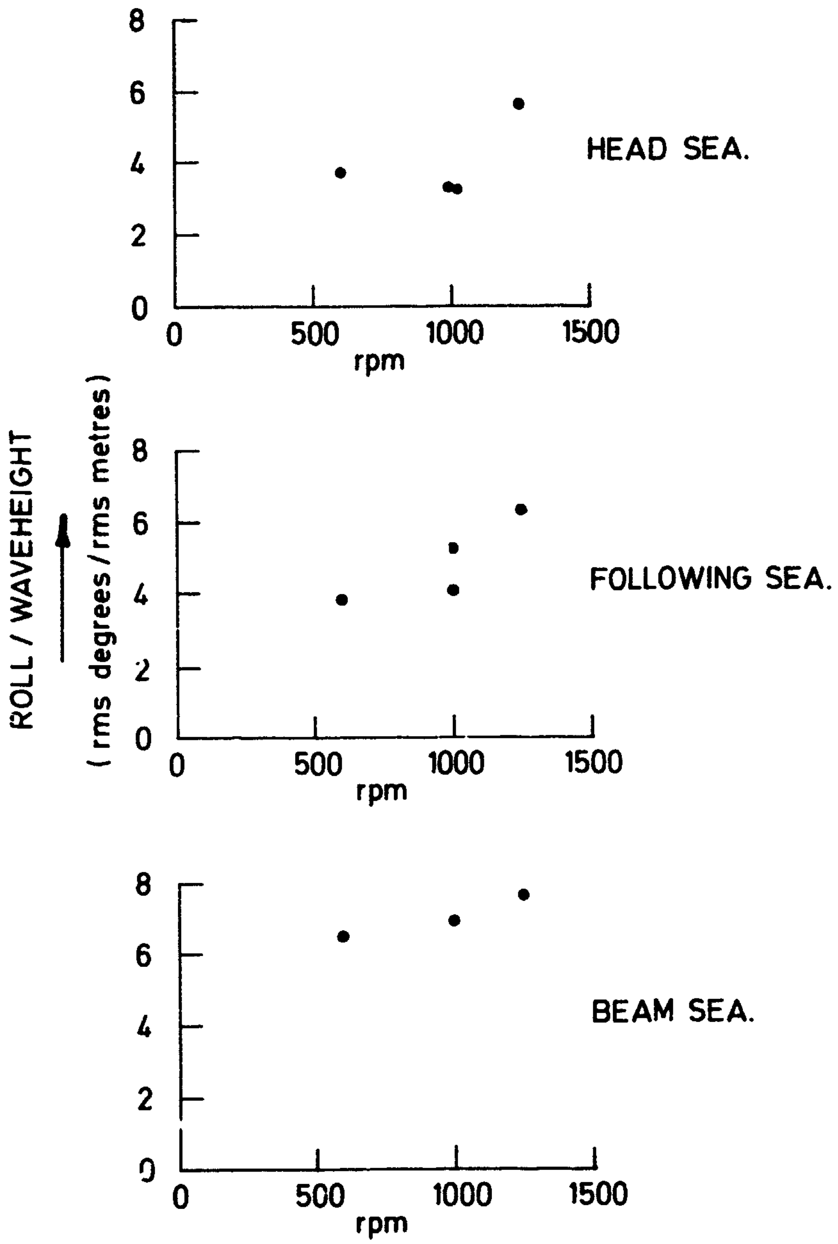


Fig. 45 Roll per unit waveheight.

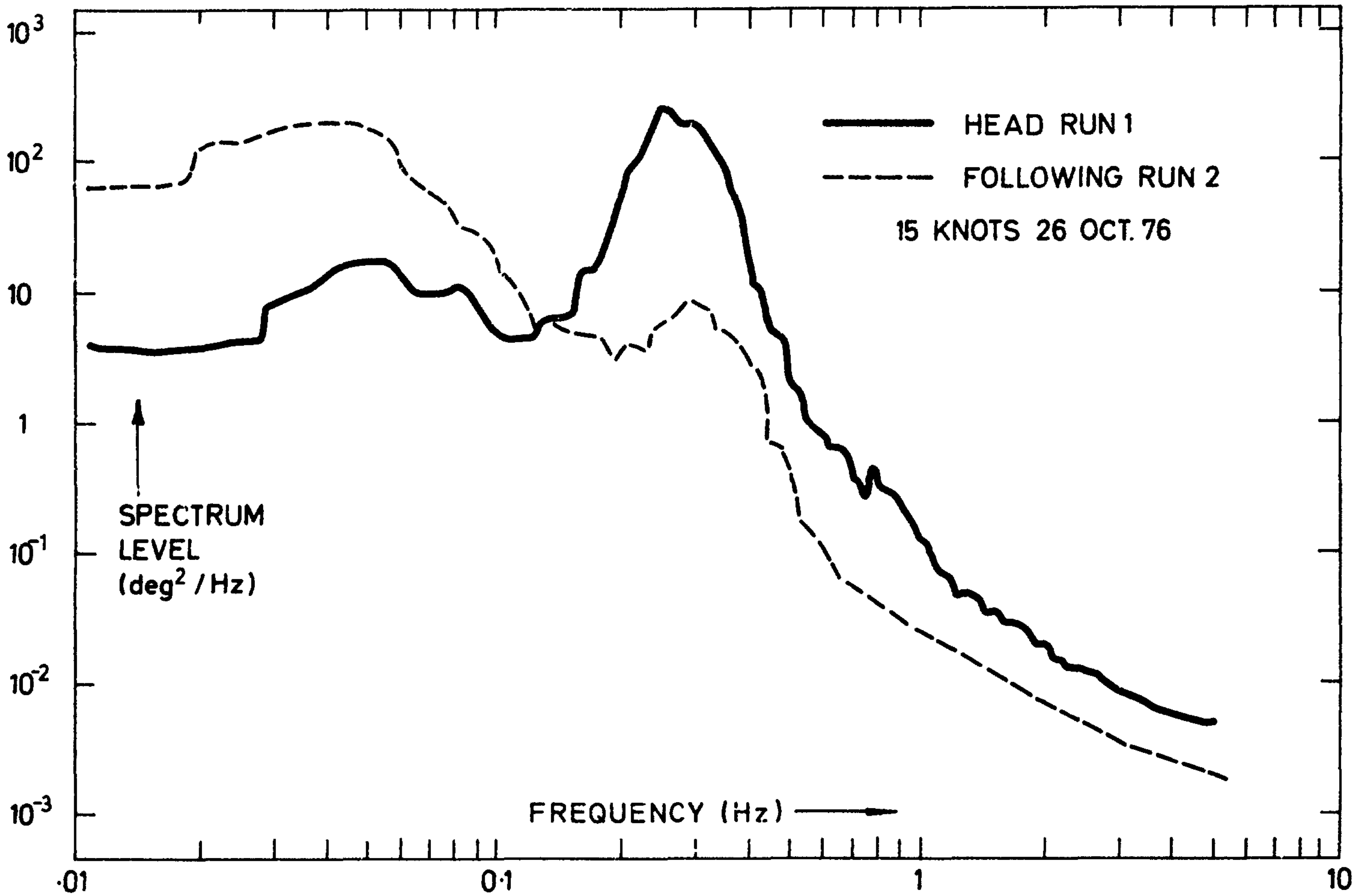


Fig. 46. Pitch spectra at 15 knots. Runs 1&2 26 Oct. 76.

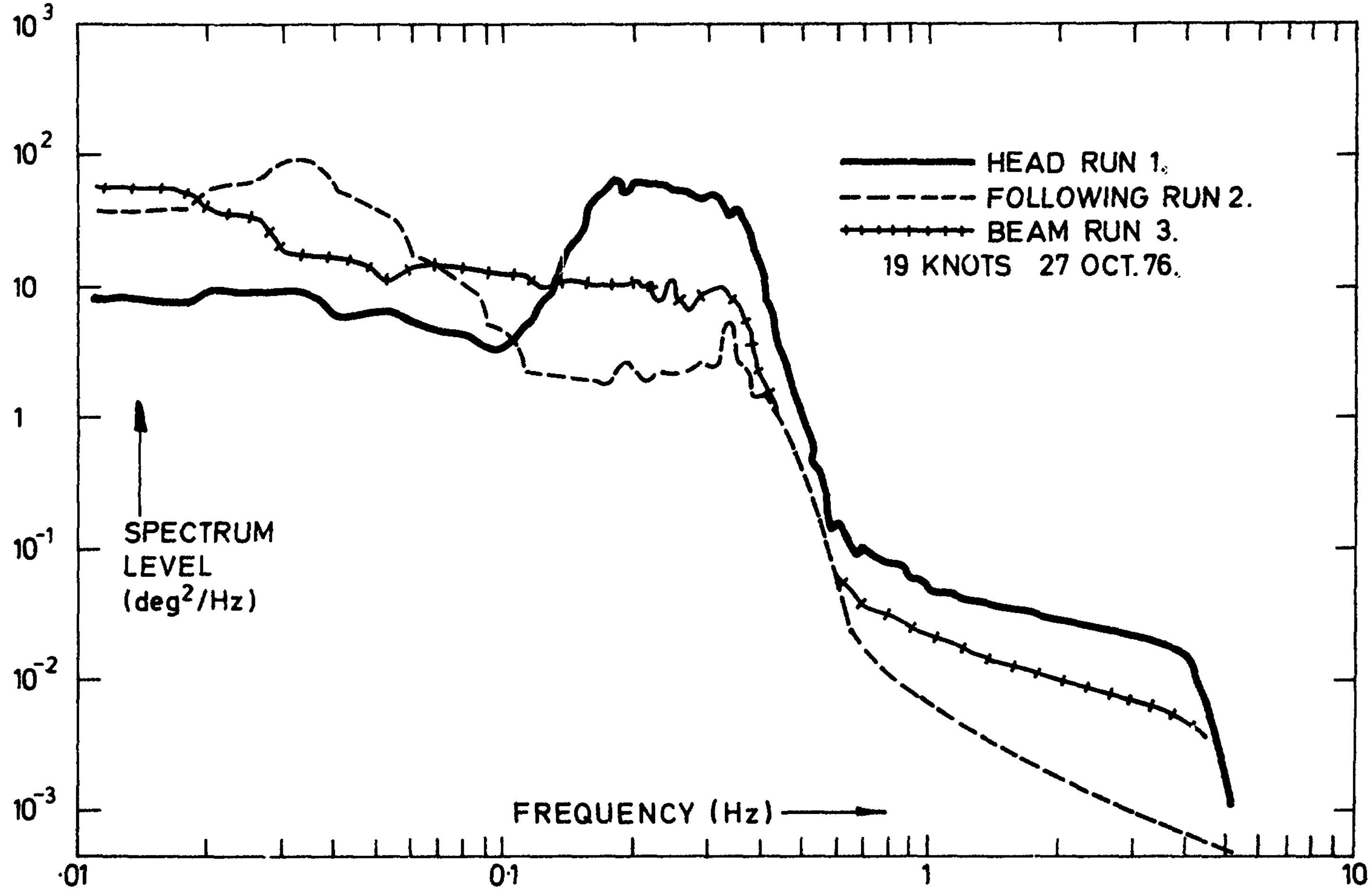


Fig. 47. Pitch spectra at 19 knots. Runs 1,2,3 27 Oct. 76.

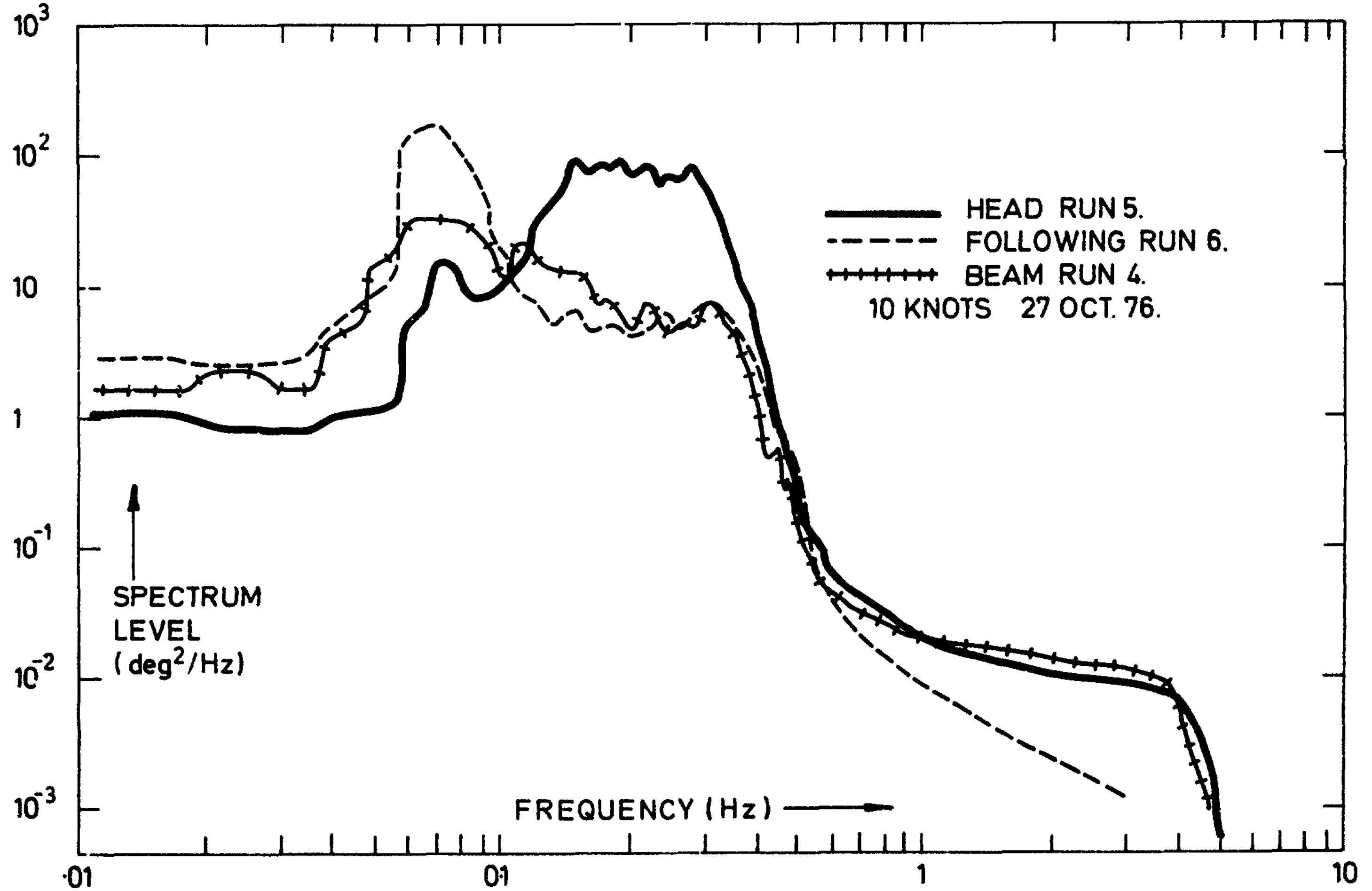


Fig. 48. Pitch spectra at 10 knots. Run 4,5,6 27 Oct. 76.

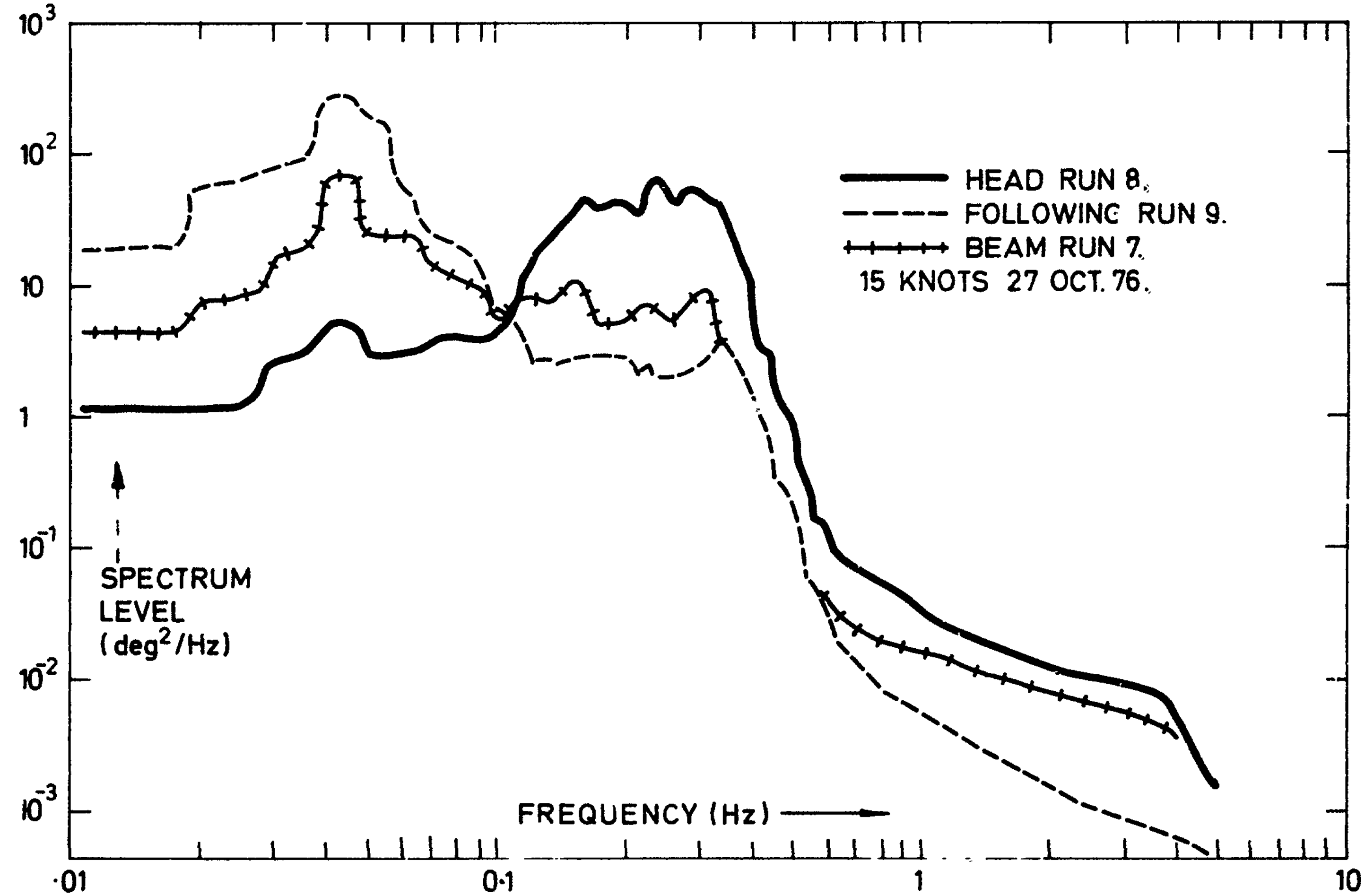


Fig.49. Pitch spectra at 15 knots. Runs 7,8,9 27 Oct. 76.

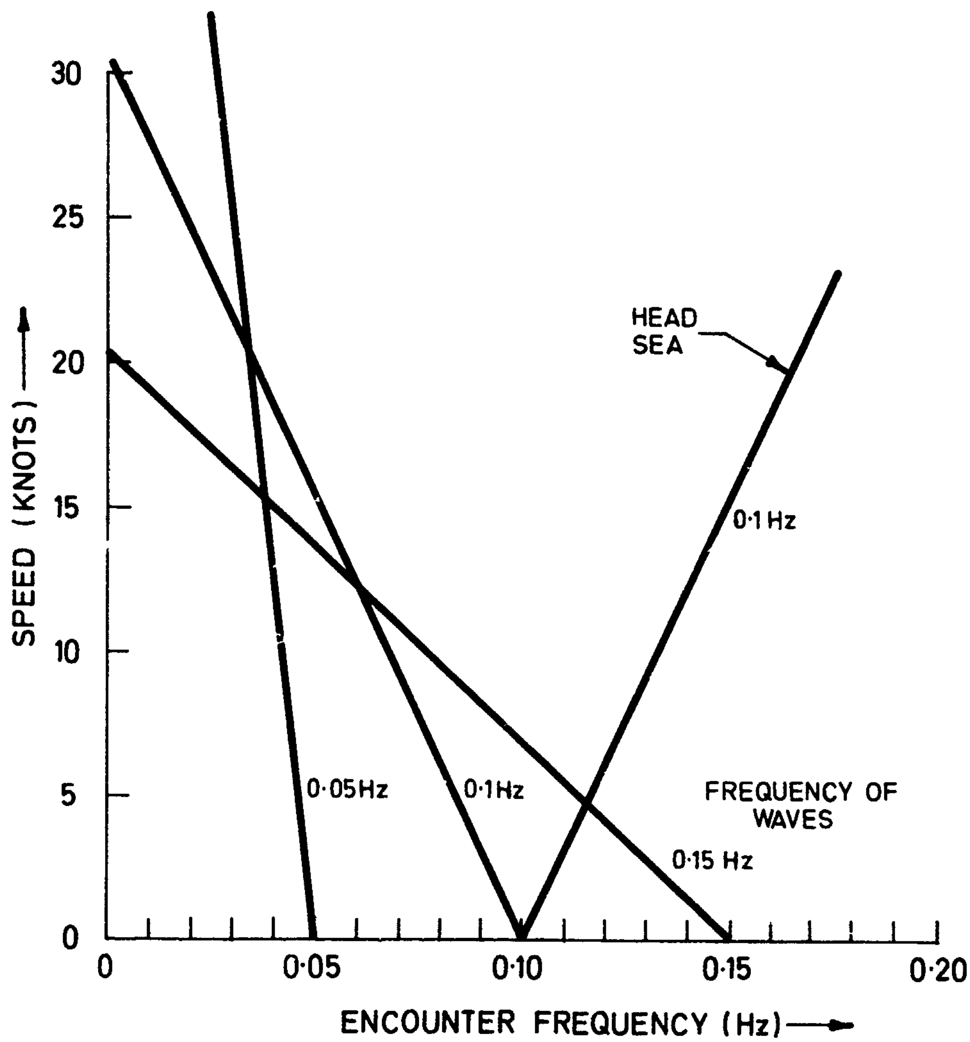


Fig. 50. Encounter frequency for an overtaking sea.

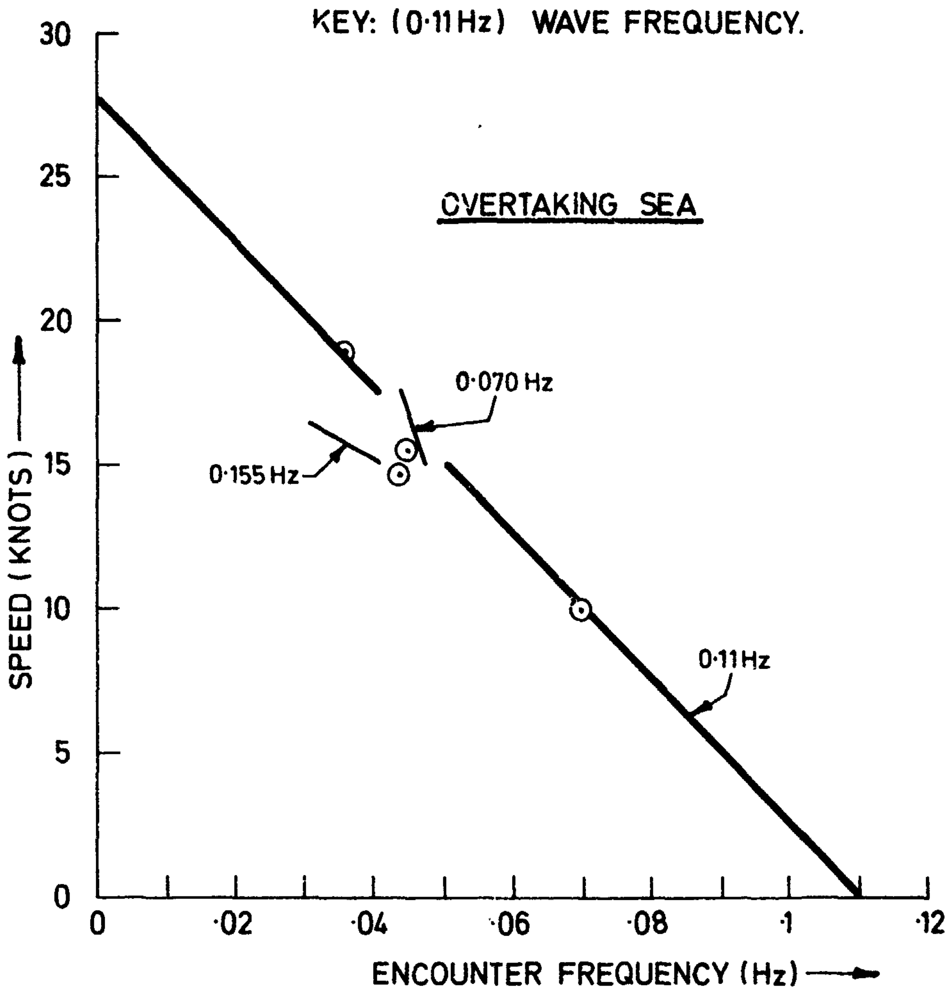


Fig. 51. Measured encounter frequency.

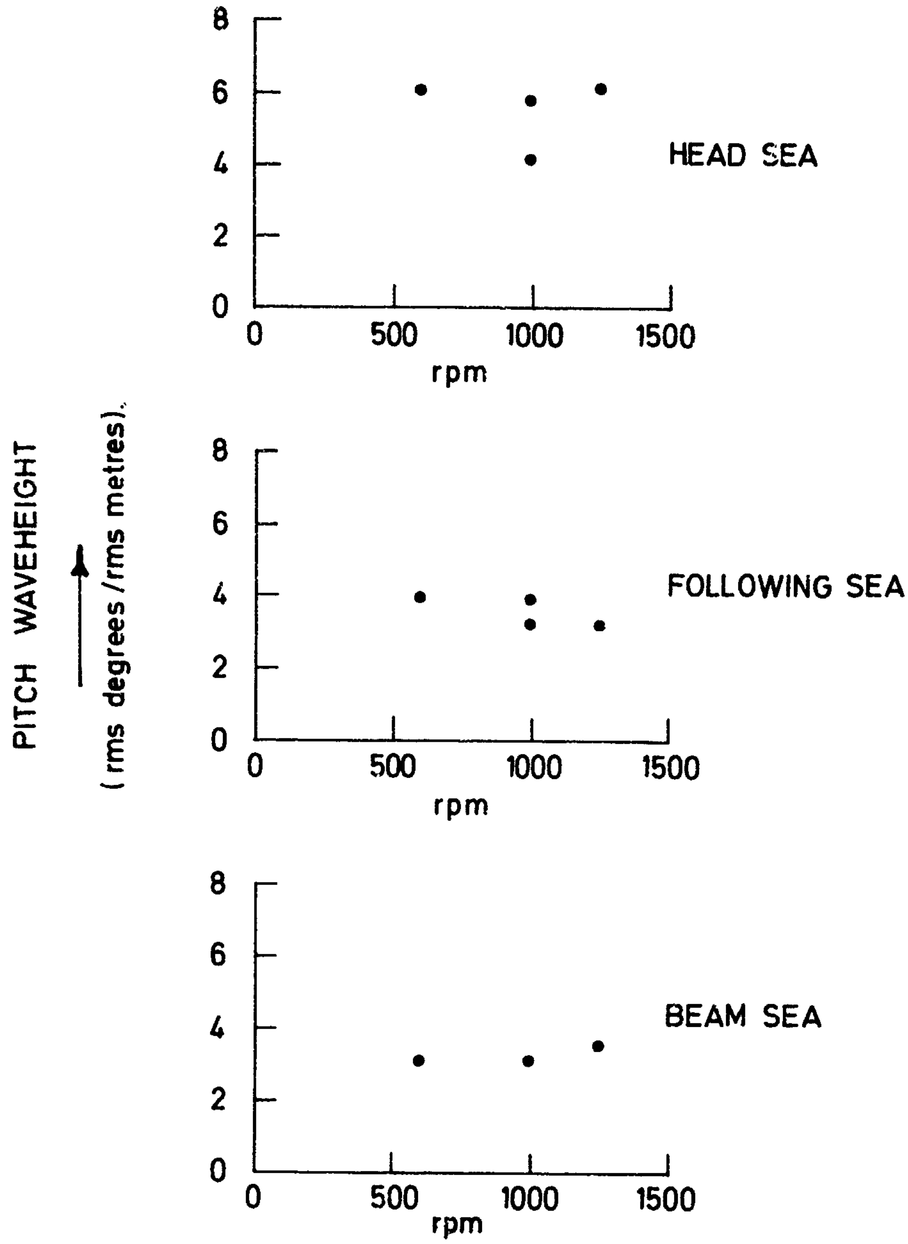


Fig.52. Pitch per unit waveheight.

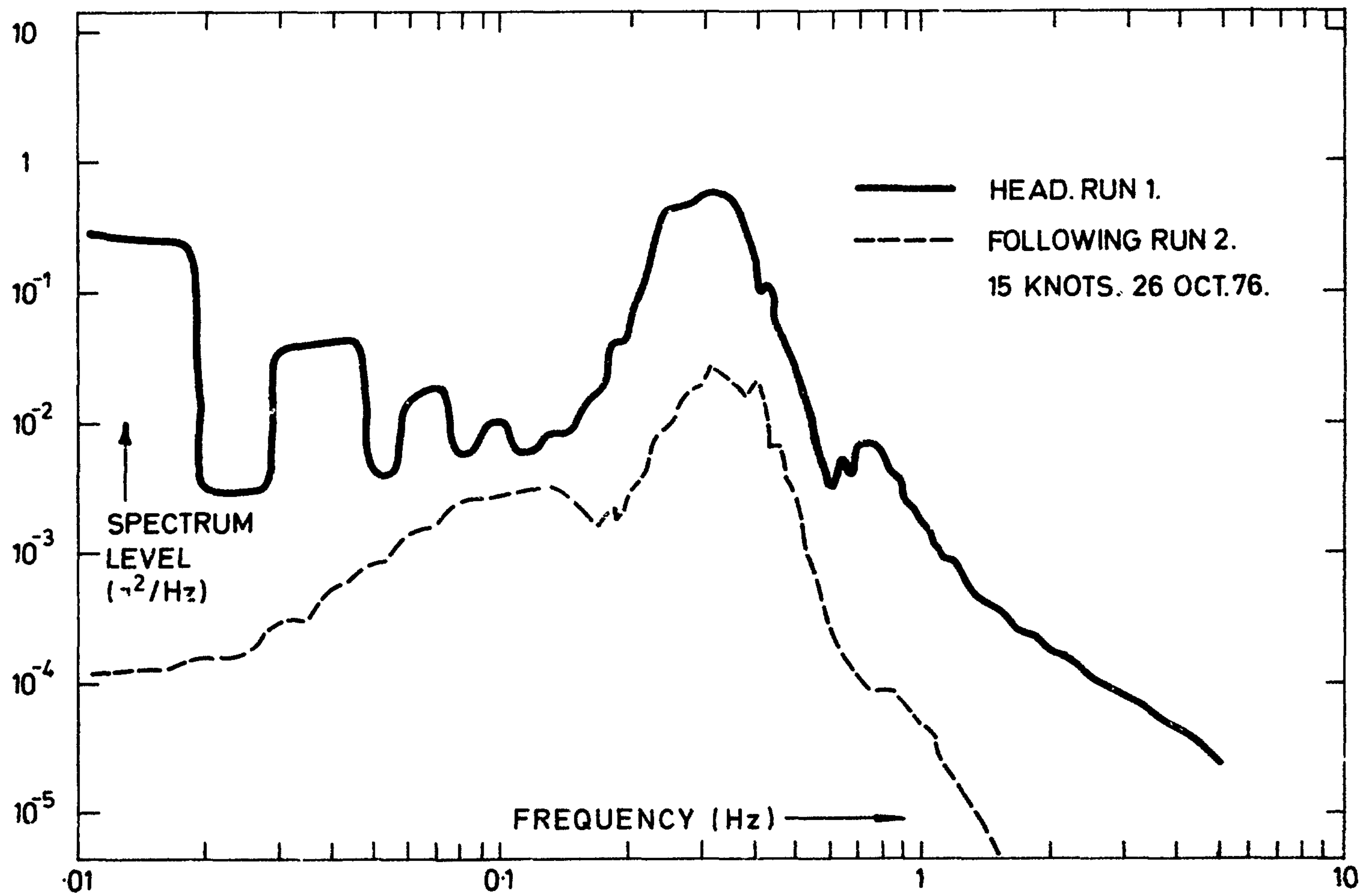


Fig.53. Vertical acceleration spectra at 15 knots. (For'd)
Runs 1 & 2 26 Oct. 76.

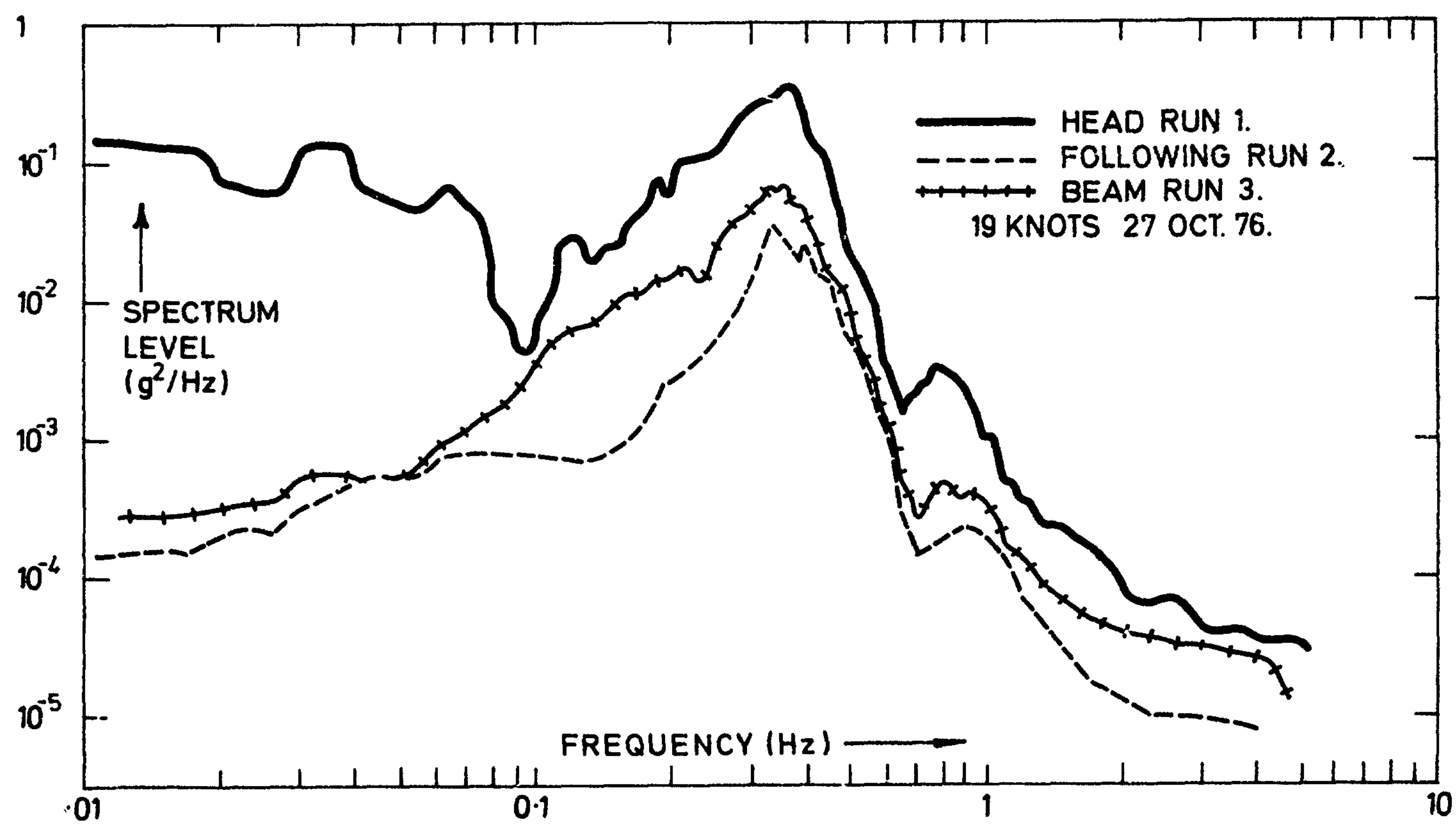


Fig.54. Vertical acceleration spectra at 19 knots (For'd) Runs 1,2,3 27 Oct.76.

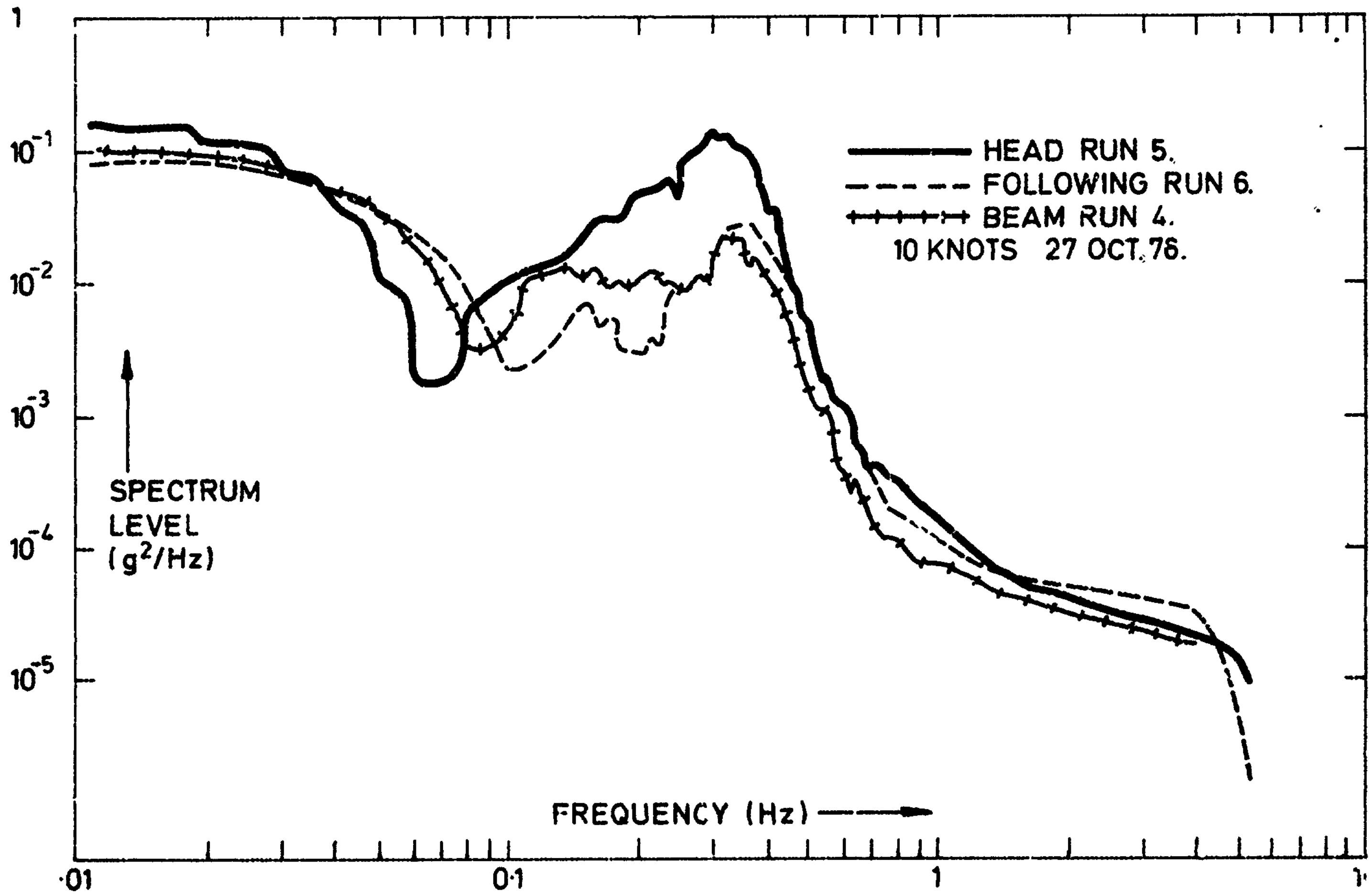


Fig.55. Vertical acceleration spectra at 10 knots (For'd)
Runs 4,5,6 27 Oct. 76.

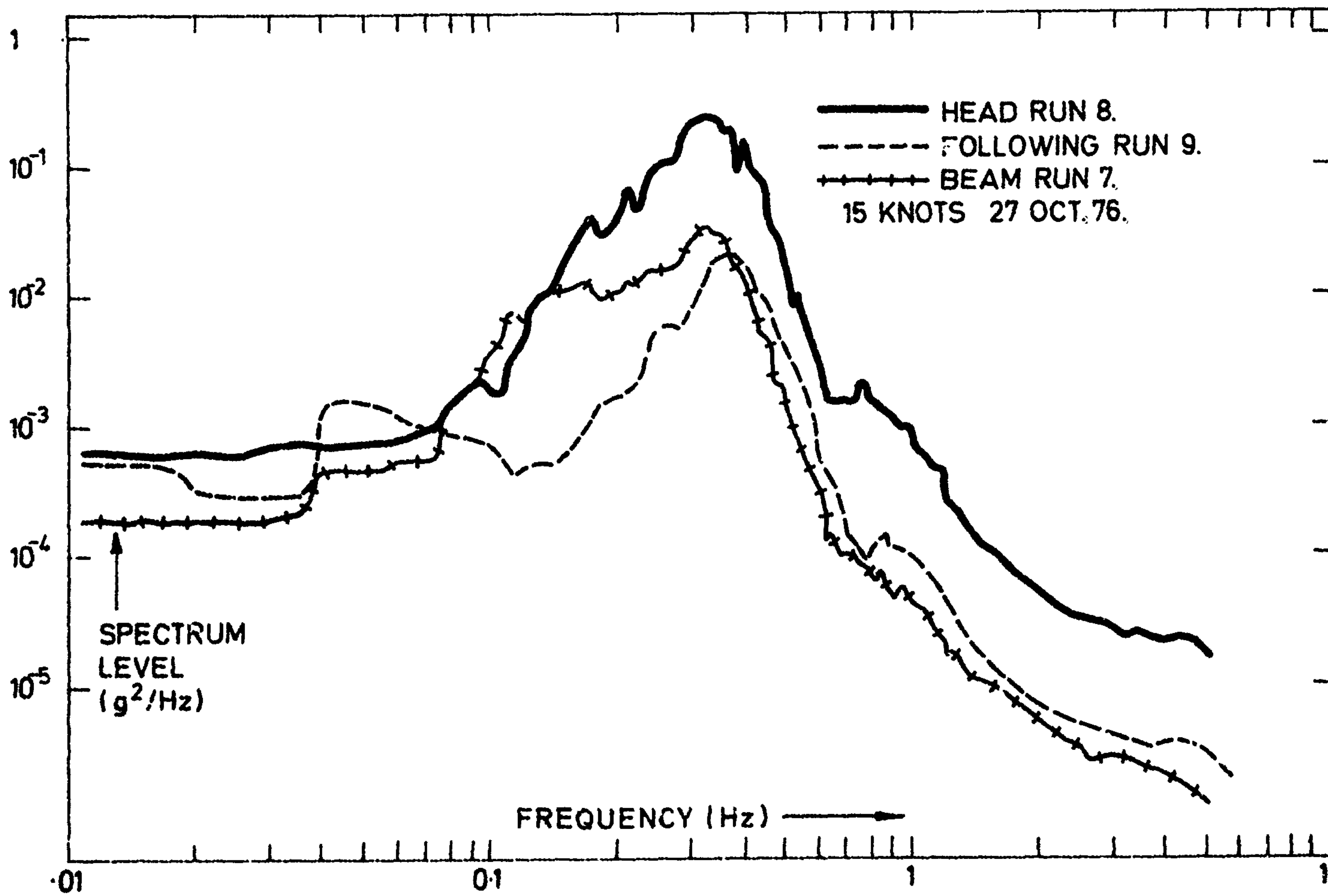


Fig.56. Vertical acceleration spectra at 15 knots (For'd)
Runs 7,8,9 27 Oct. 76.

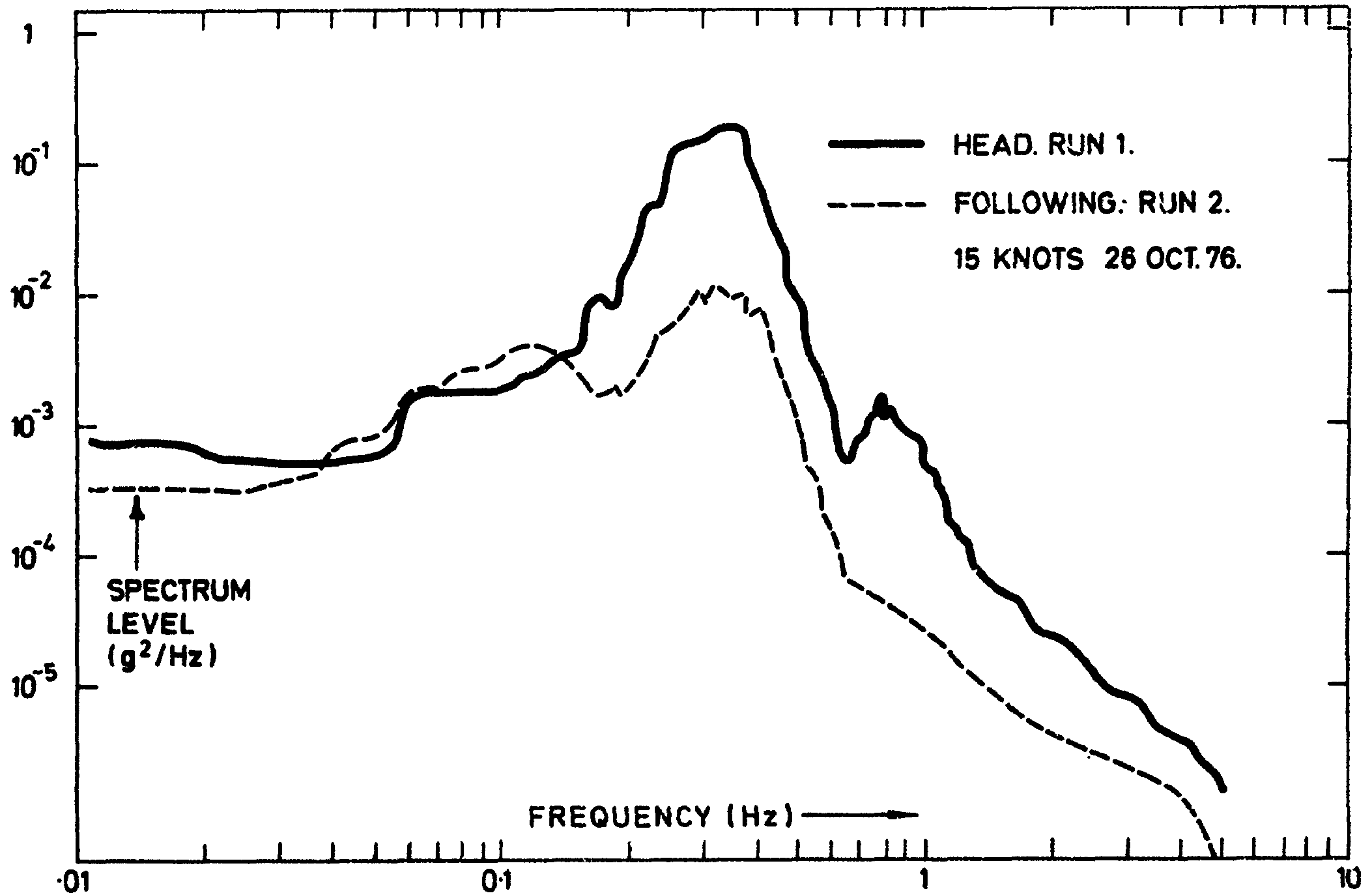


Fig. 57. Vertical acceleration spectra at 15 knots (CO's Cabin). Runs 1 & 2 26 Oct. 76.

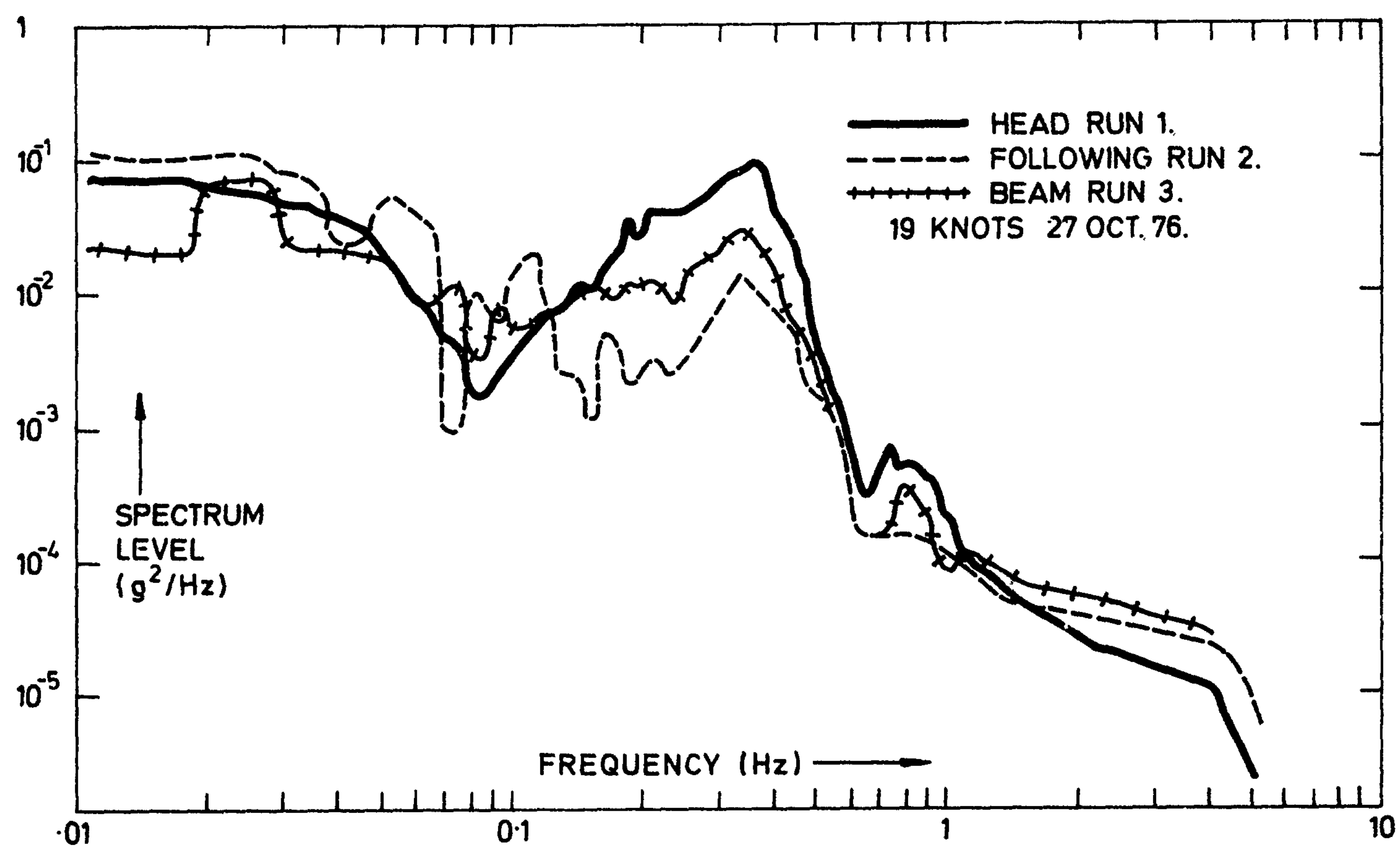


Fig.58. Vertical acceleration spectra at 19 knots. (CO's Cabin). Runs 1,2,3 27 Oct. 76.

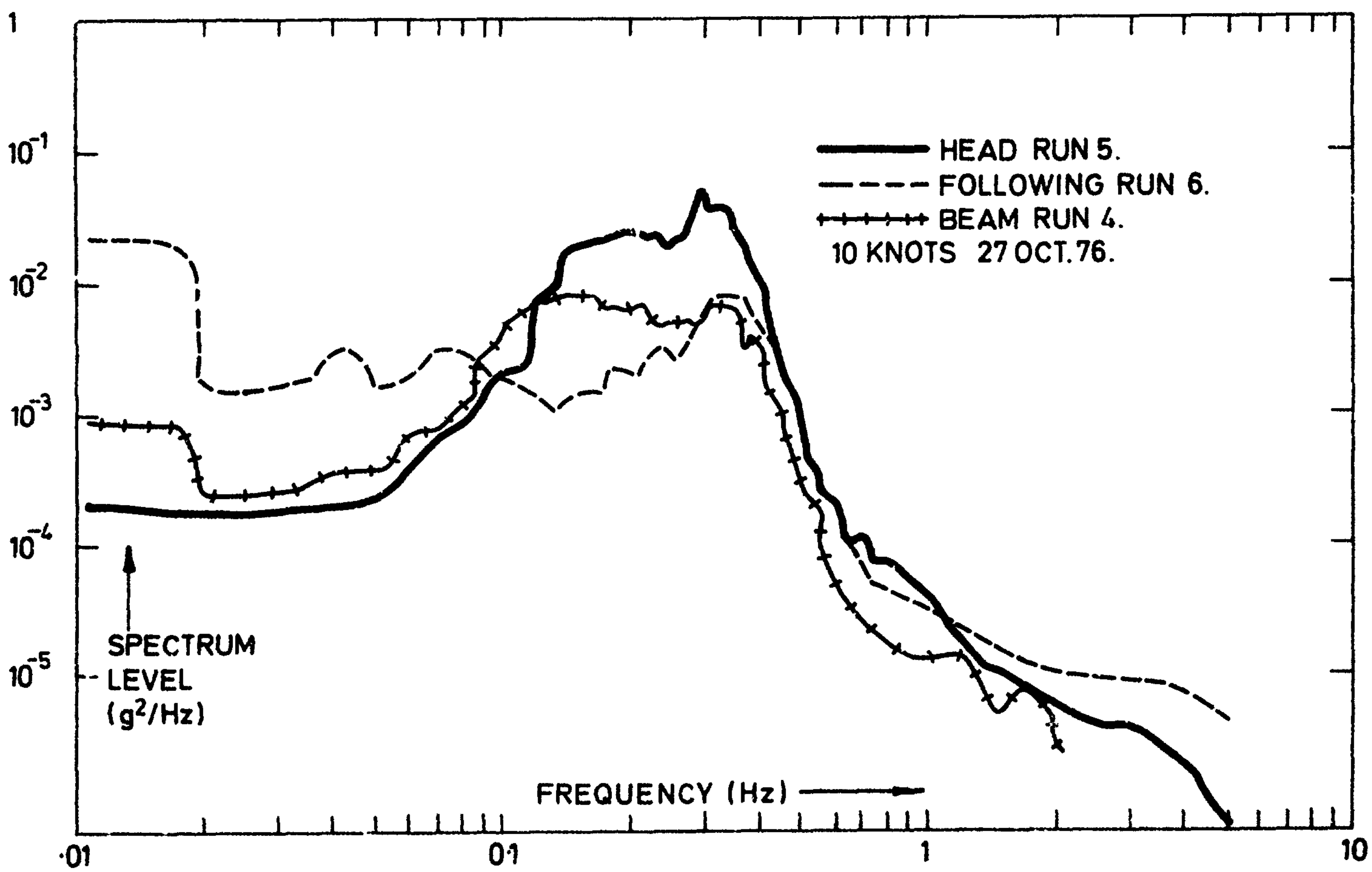


Fig. 59. Vertical acceleration spectra at 10 knots. (CO's Cabin) Runs 4,5,6 27 Oct. 76.

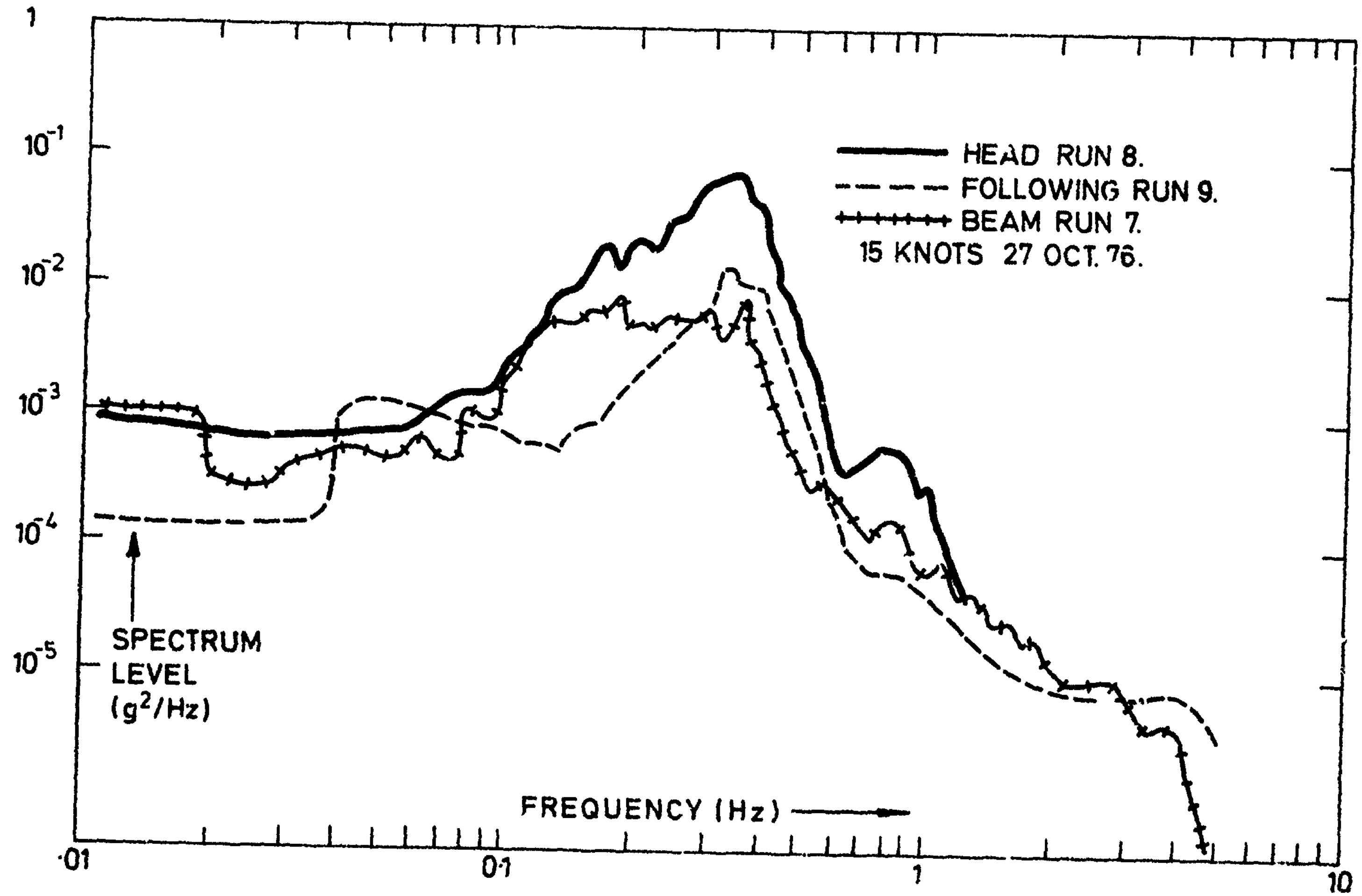


Fig. 60. Vertical acceleration spectra at 15 knots (CO's Cabin) Runs 7,8,9 27 Oct. 76.

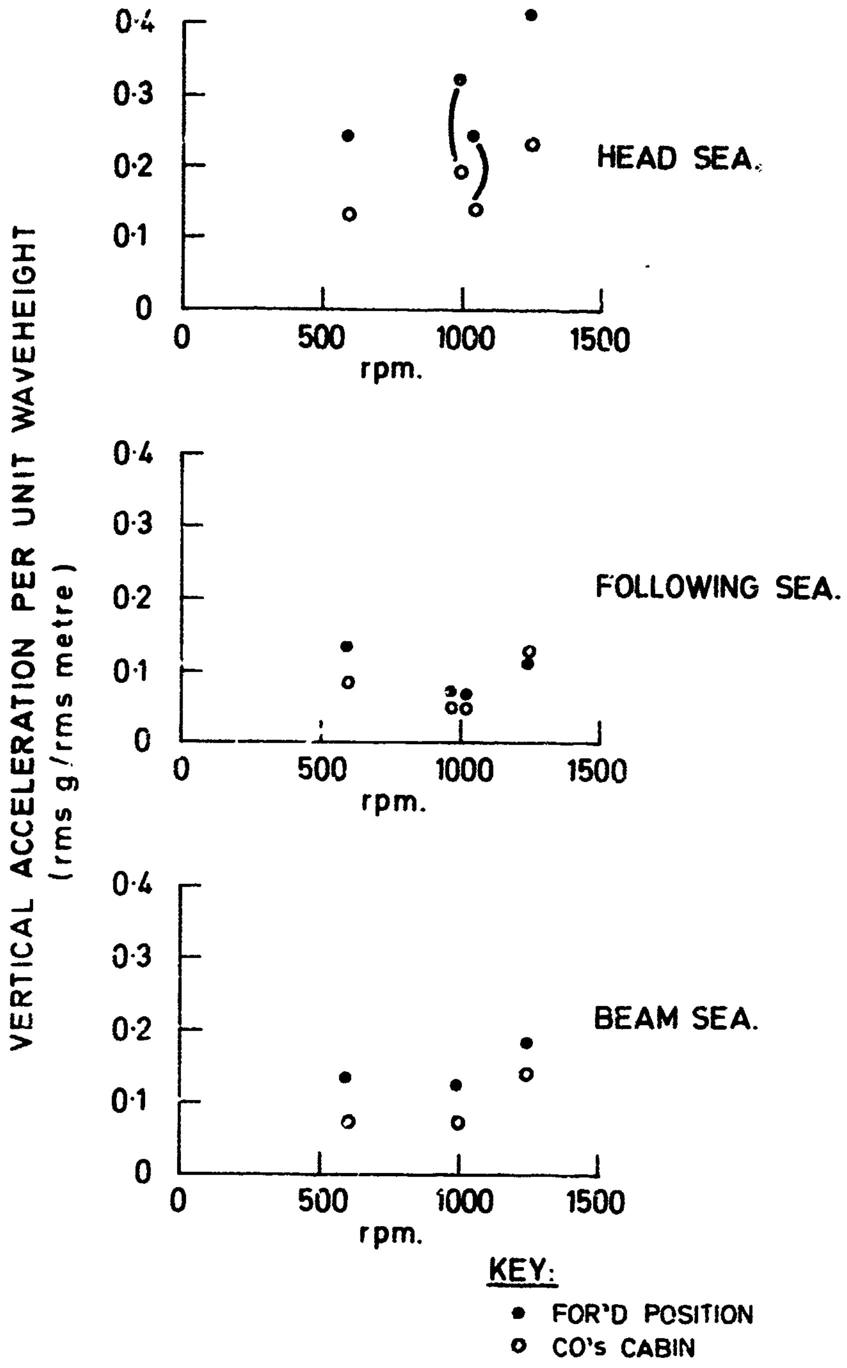


Fig. 61. Vertical acceleration per unit waveheight.

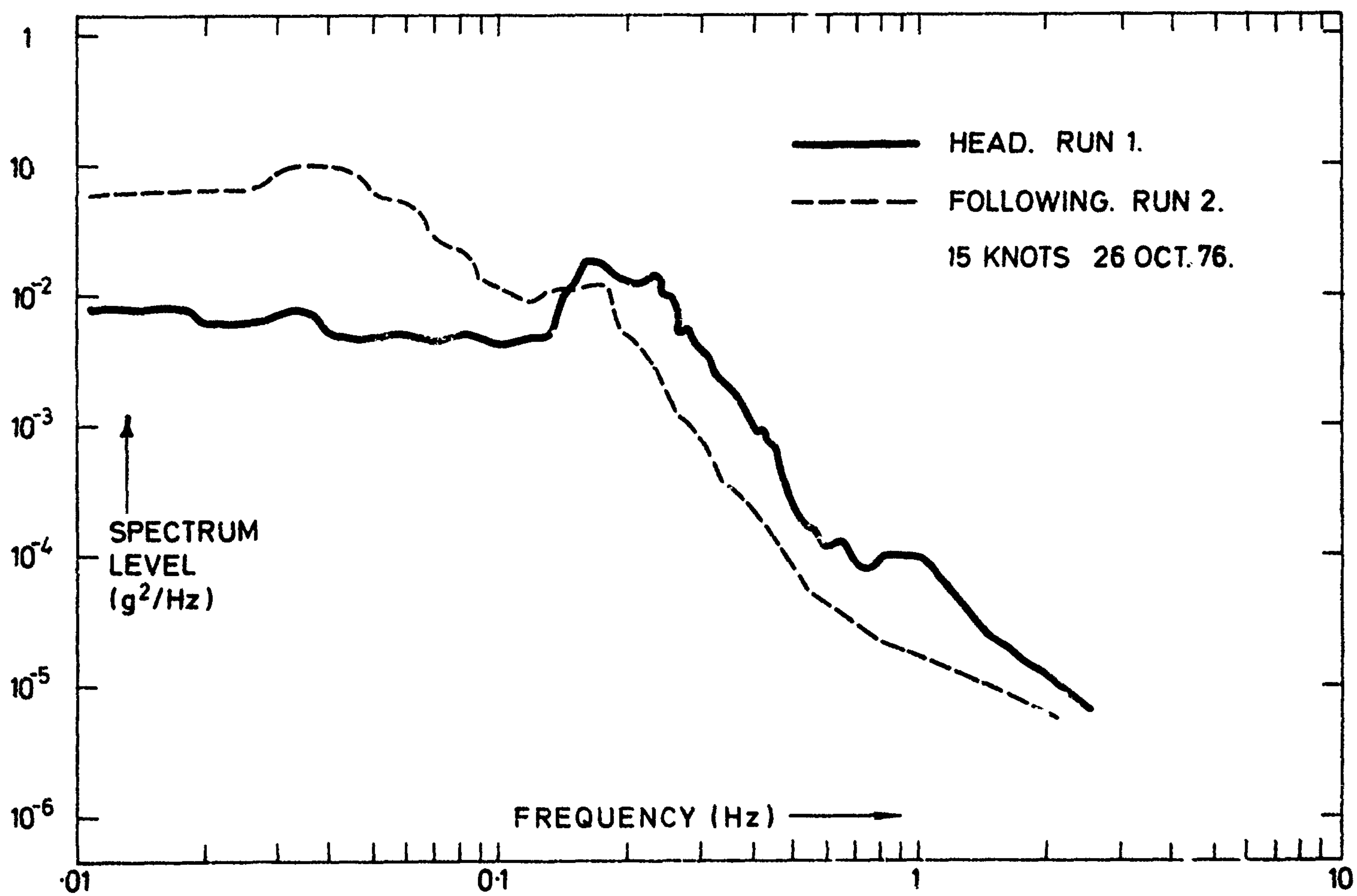


Fig.62. Lateral acceleration spectra at 15 knots (CO's cabin)
Runs 1 & 2 26 Oct. 76.

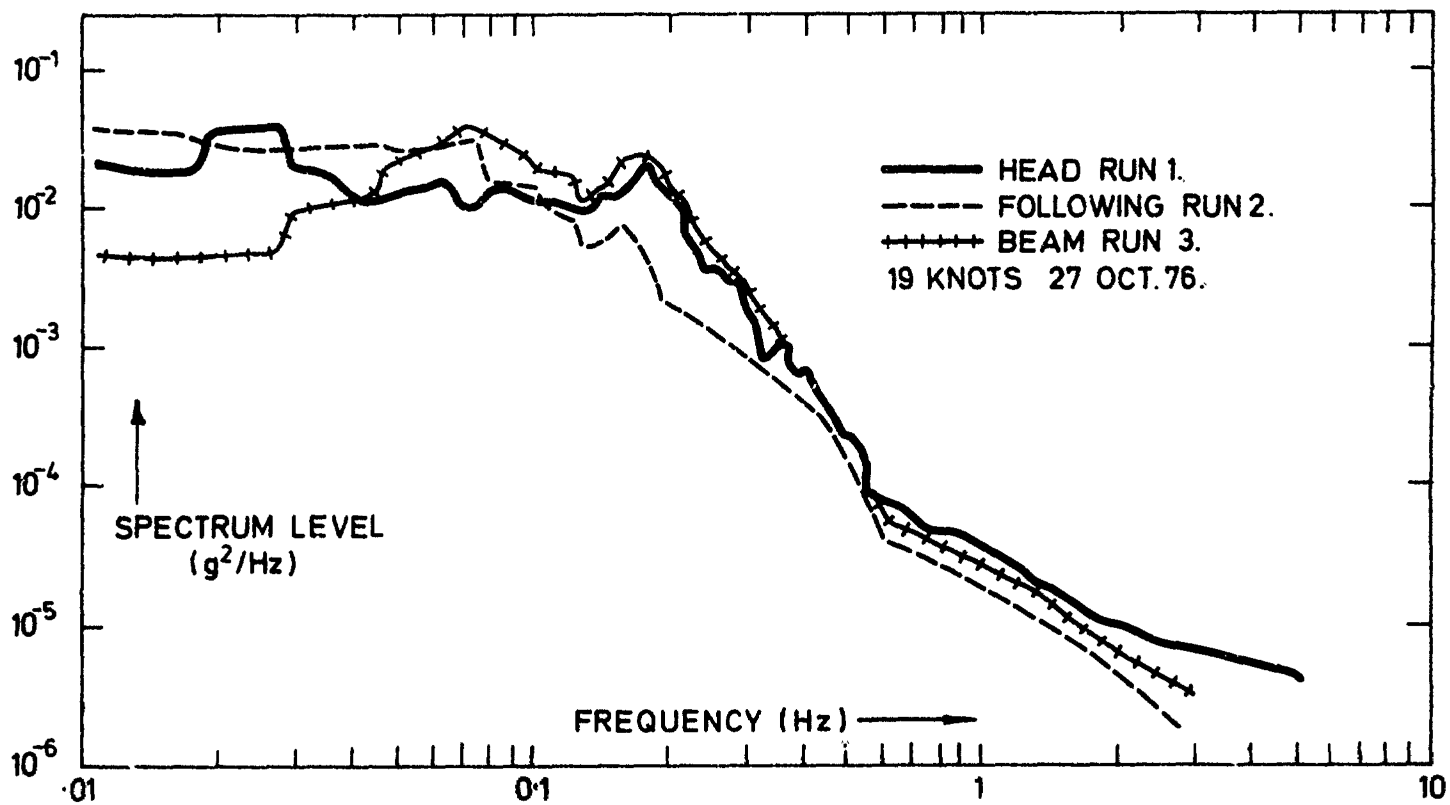


Fig. 63. Lateral acceleration spectra at 19 knots.
(CO's Cabin) Runs 1,2,3 27 Oct. 76.

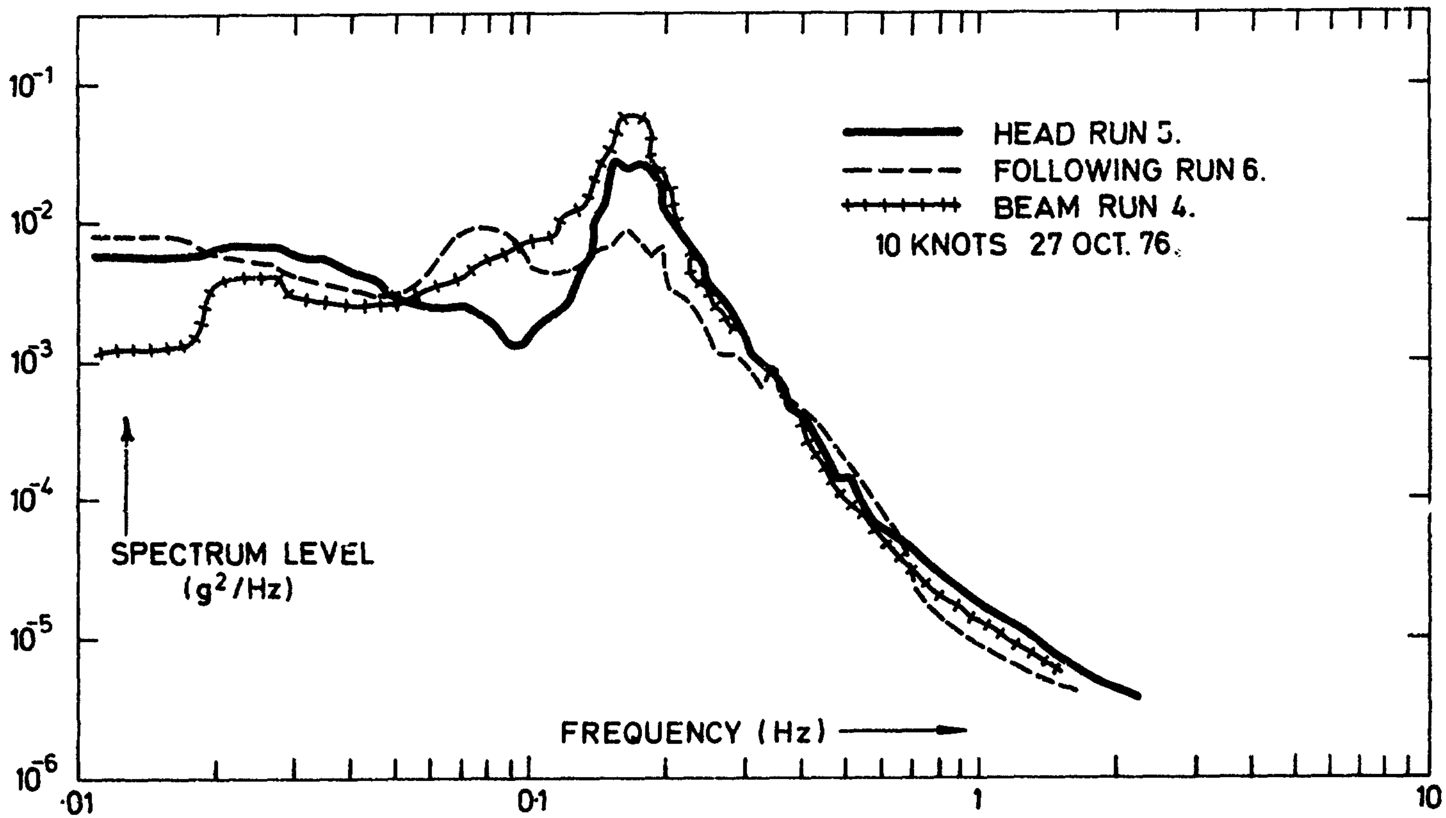


Fig. 64. Lateral acceleration spectra at 10 knots. (CO's Cabin) Runs 4,5,6 27 Oct. 76.

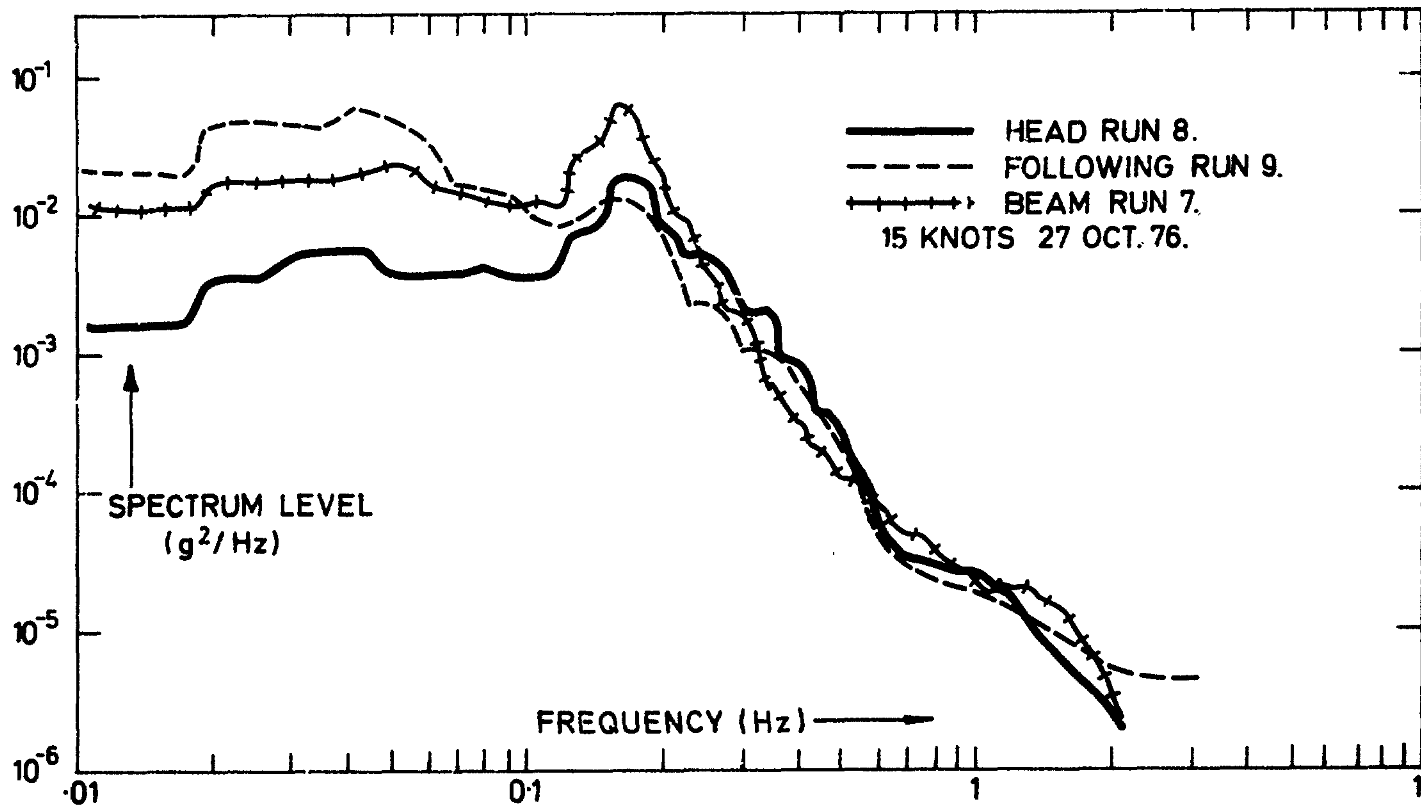


Fig.65. Lateral acceleration spectra at 15 knots. (CO's Cabin). Runs 7,8,9 27 Oct. 76.

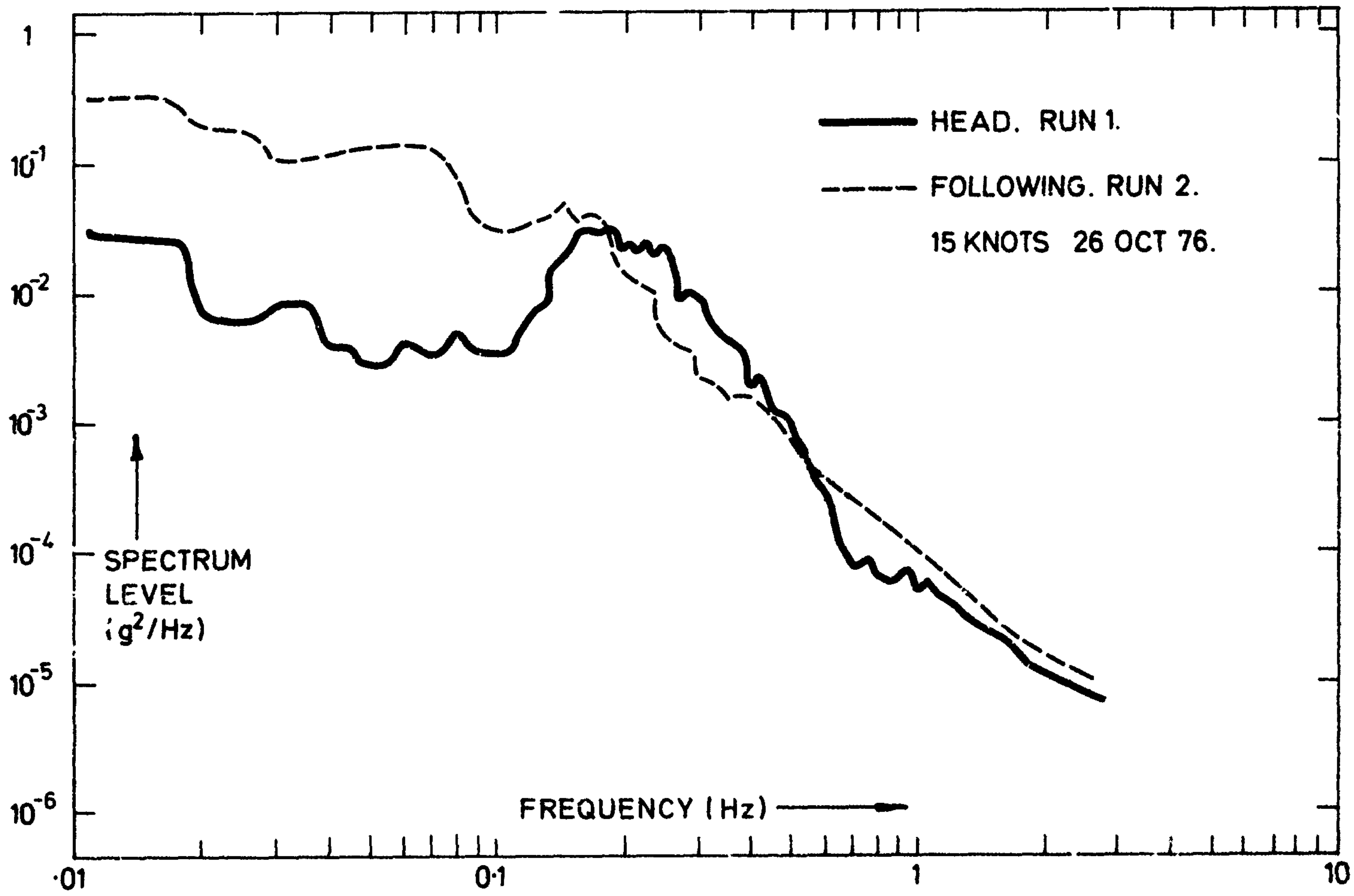


Fig. 66. Lateral acceleration spectra at 15 knots (Wheelhouse)
Runs 1,2 26 Oct. 76.

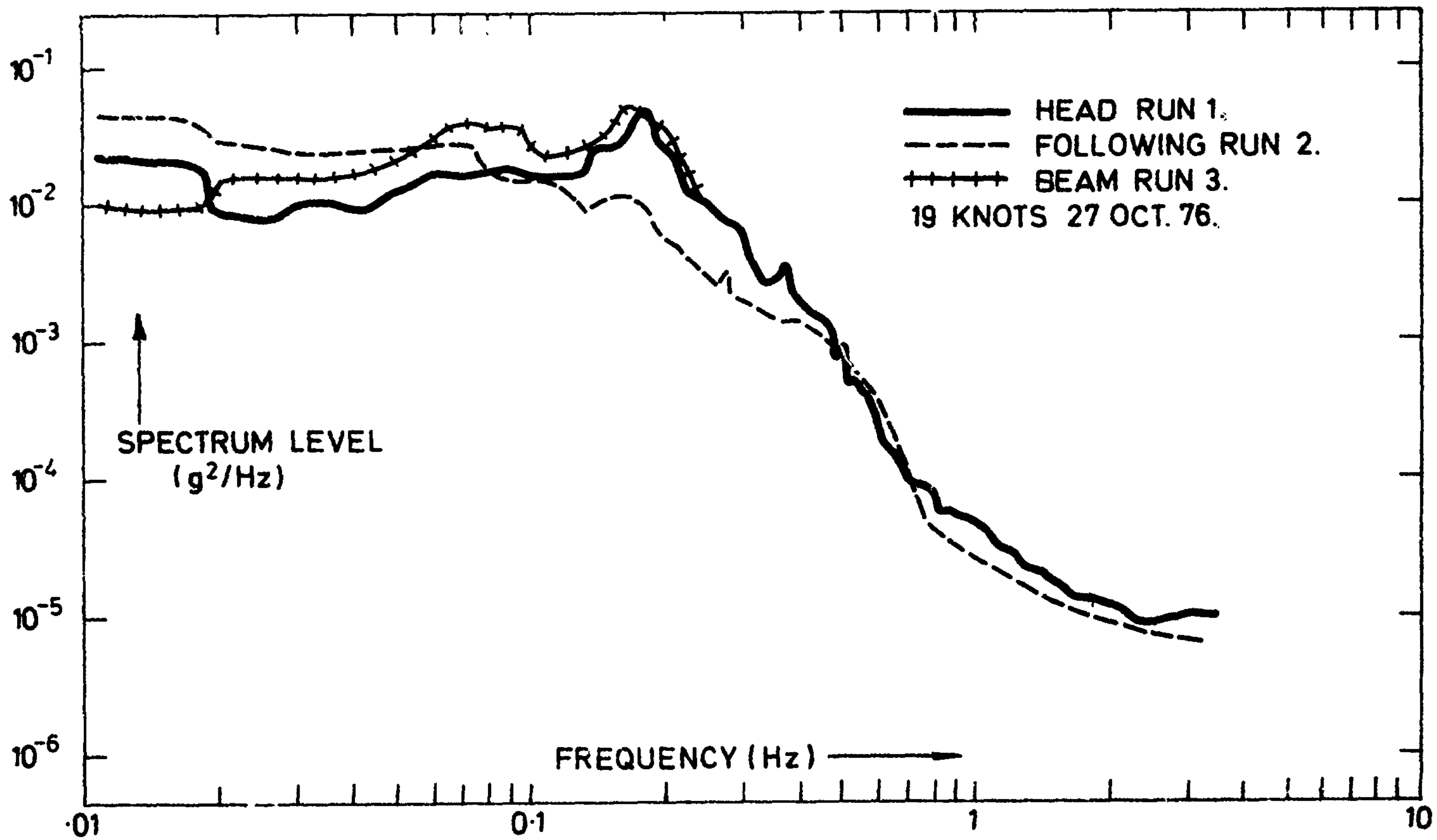


Fig. 67. Lateral acceleration spectra at 19 knots. (Wheelhouse) Runs 1,2,3 27 Oct. 76.

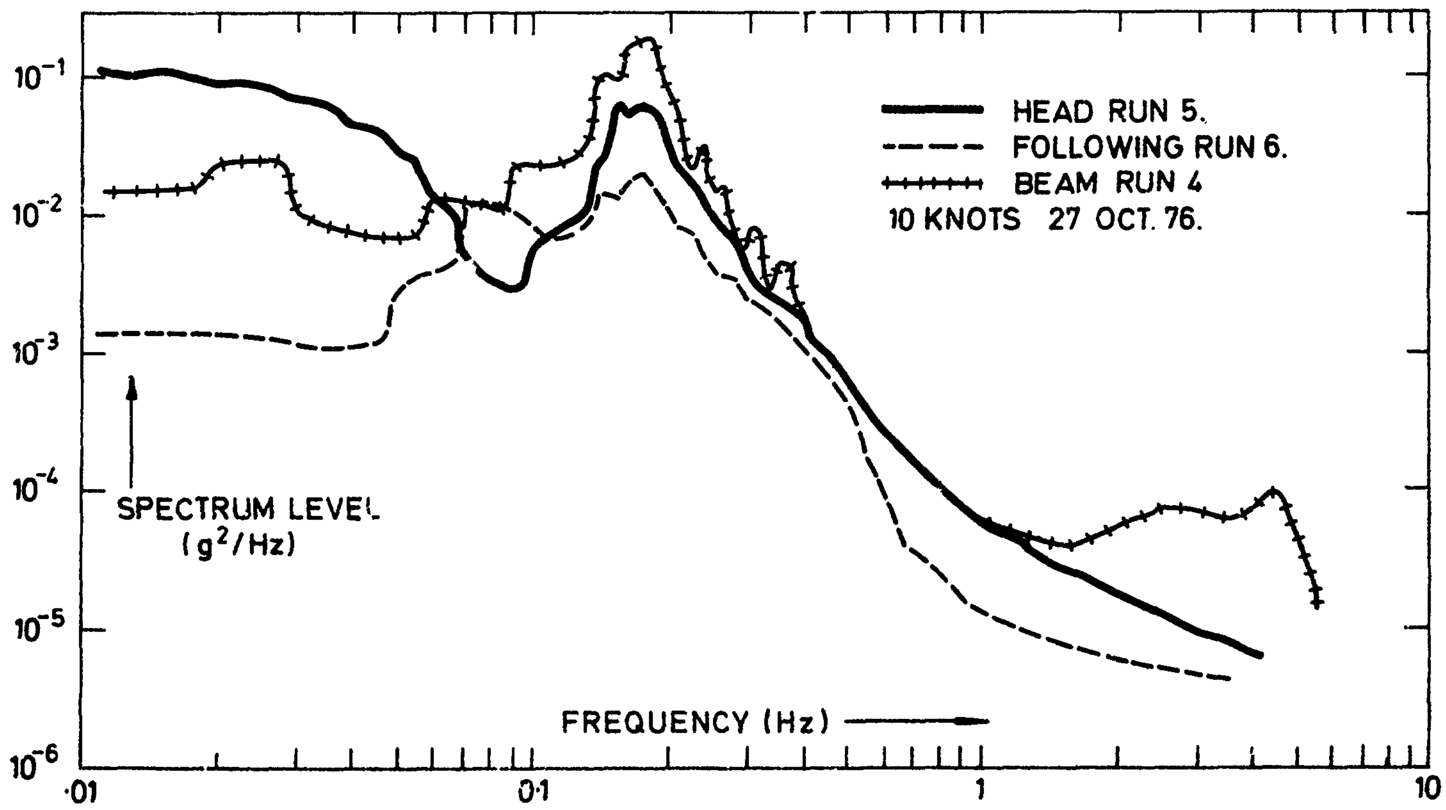


Fig. 68. Lateral acceleration spectra at 10 knots.
(Wheelhouse) Runs 4,5,6 27 Oct. 76.

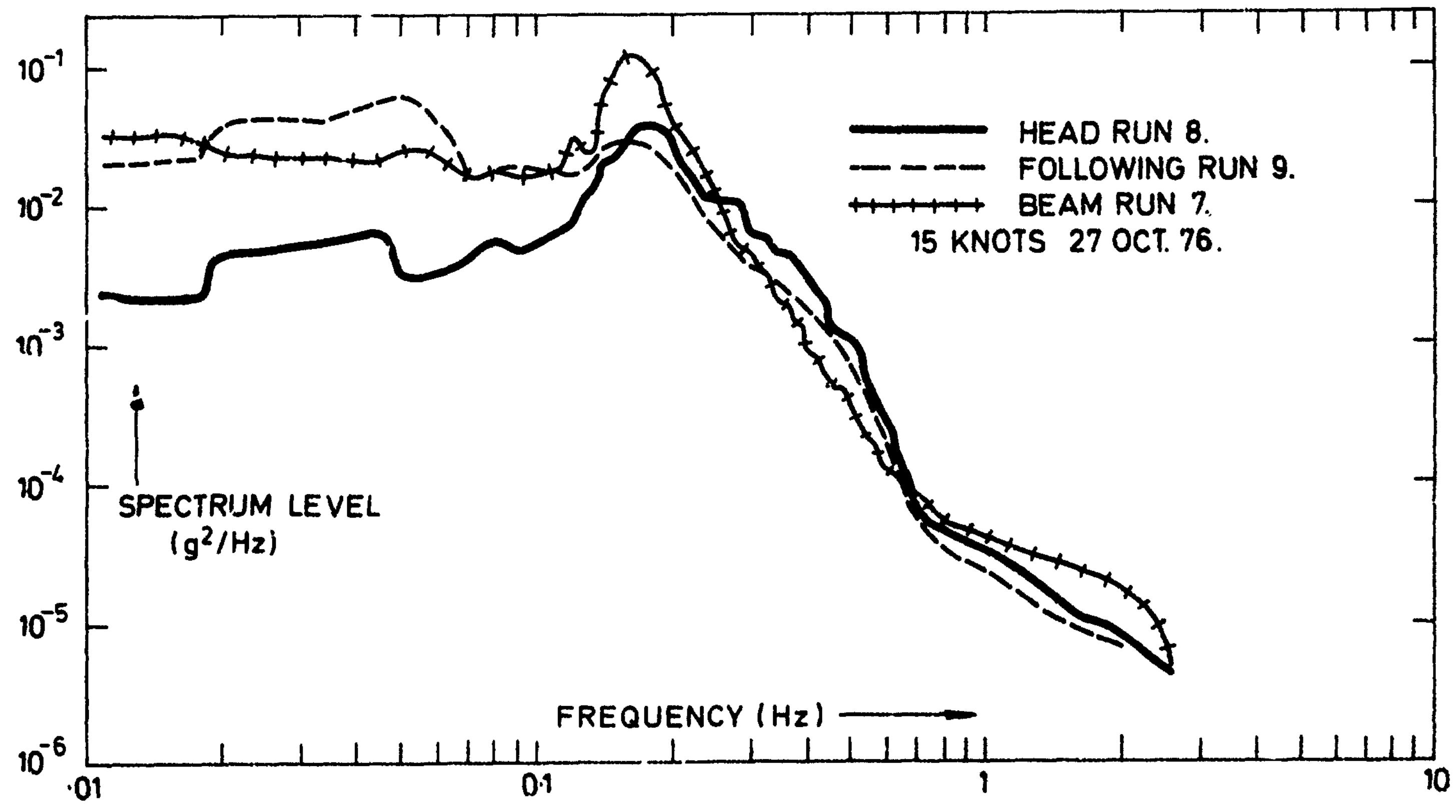


Fig. 69. Lateral acceleration spectra at 15 knots. (Wheelhouse) Runs 7,8,9 27 Oct. 76.

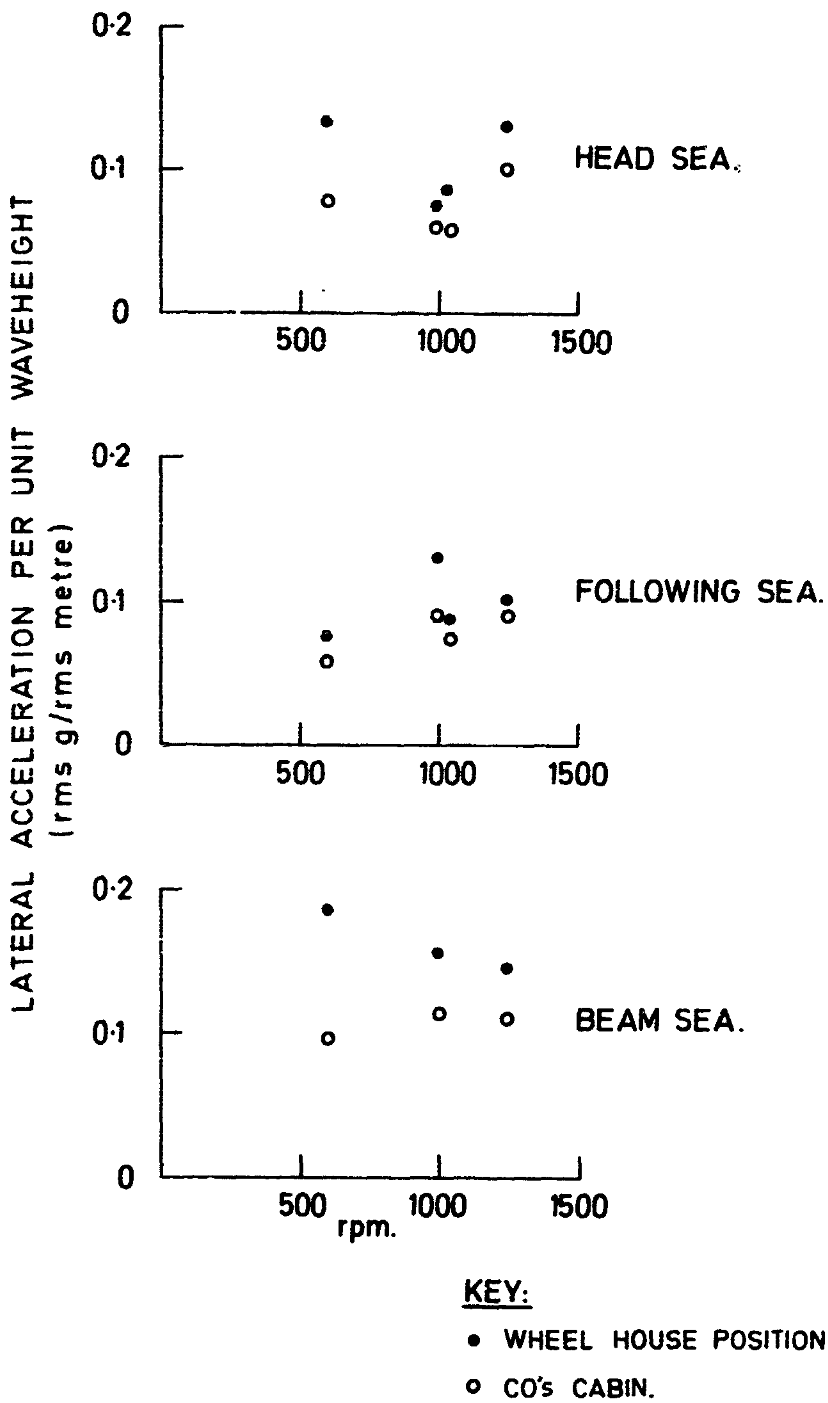
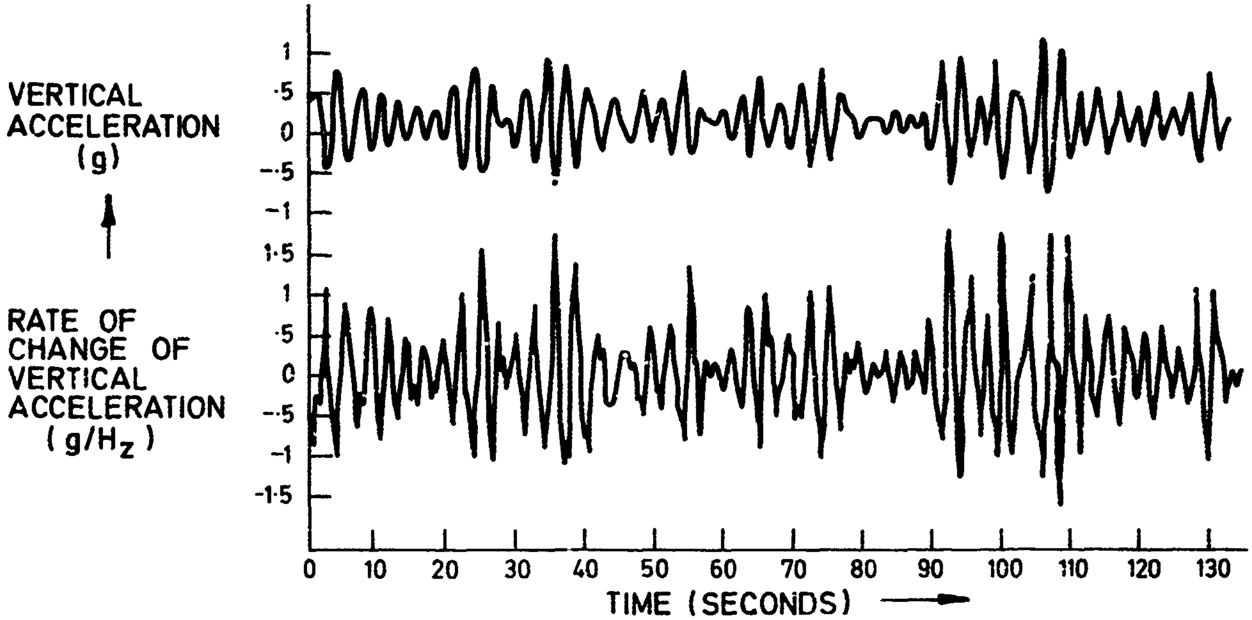
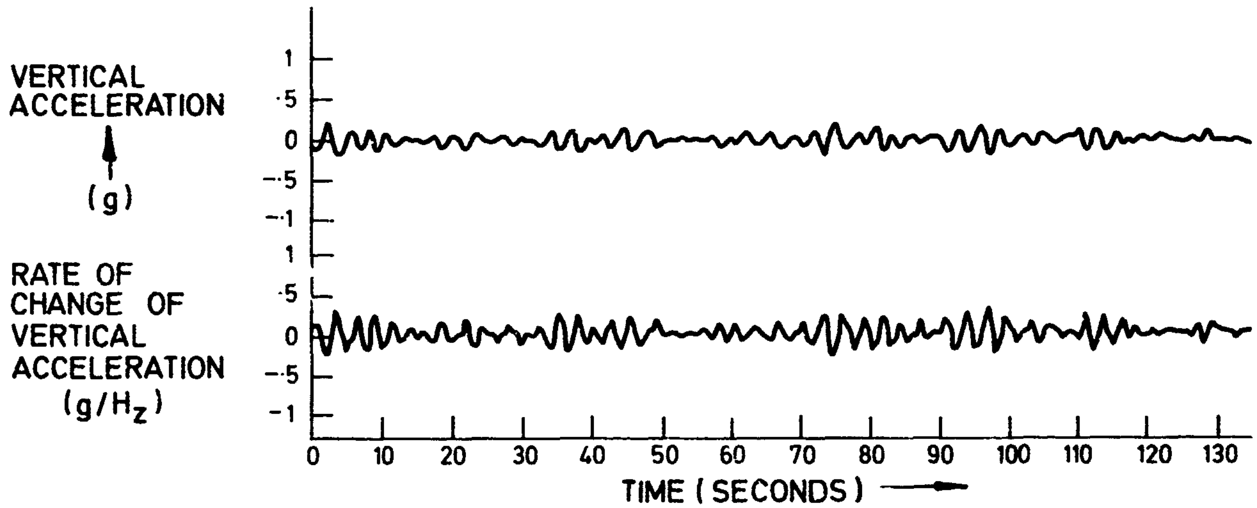


Fig.70. Lateral acceleration per unit waveheight.

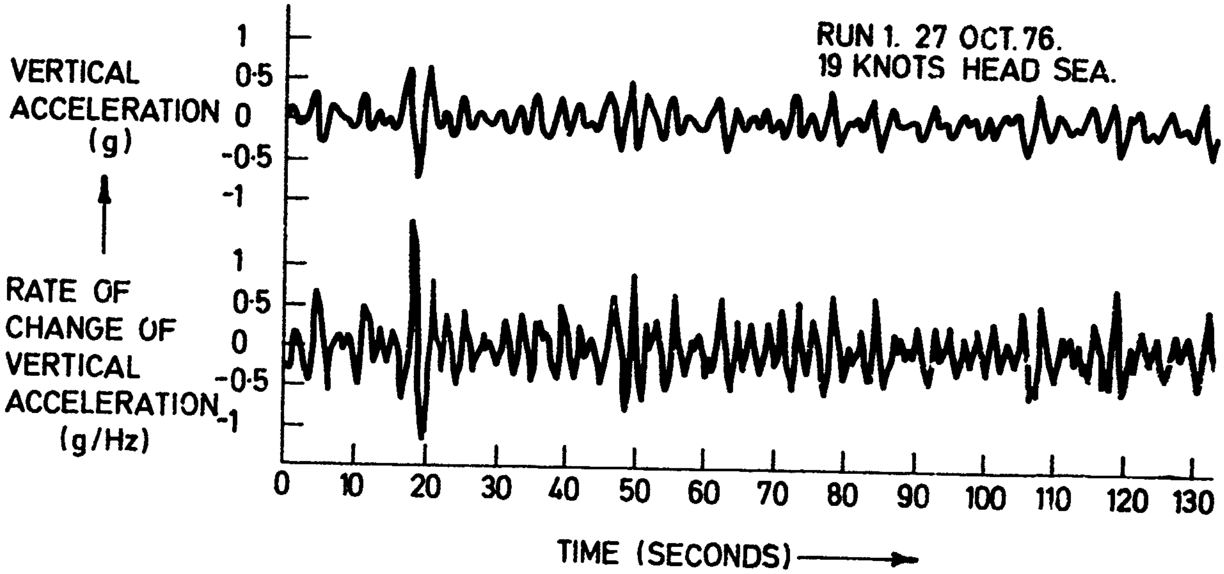


RUN 1. 26 OCT'76. 15 KNOTS. HEAD SEA.



RUN 2. 26 OCT'76. 15 KNOTS. FOLLOWING SEA.

Fig.71. Vertical acceleration (For'd) and its derivative Runs 1& 2 26 OCT'76.



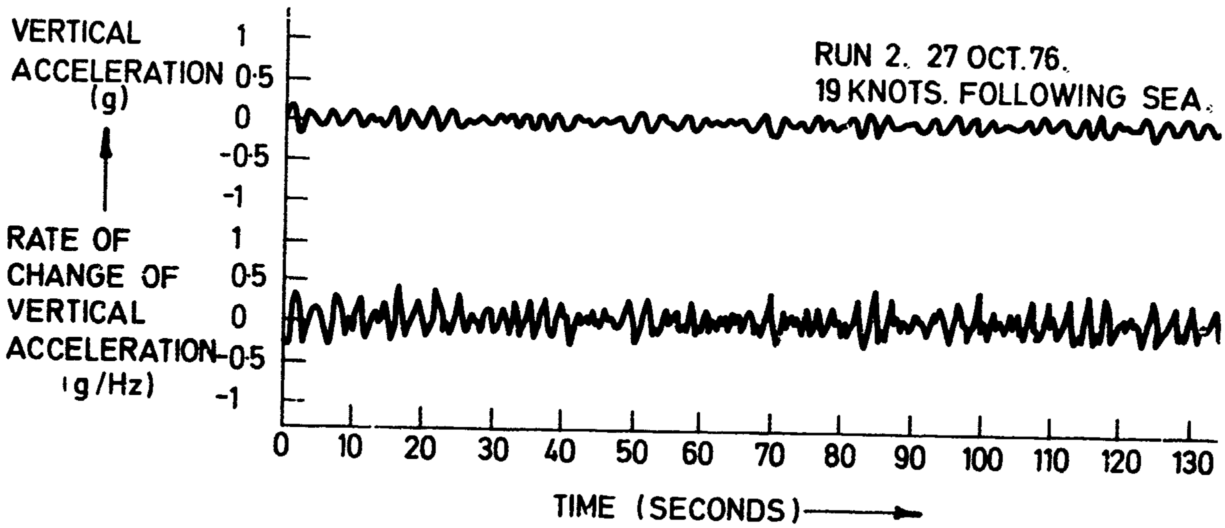
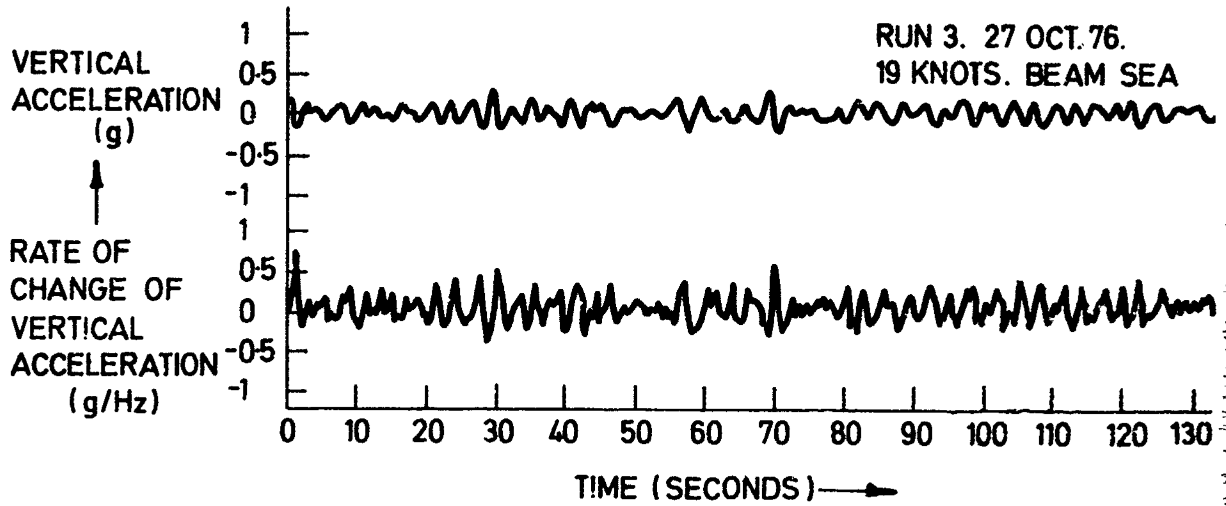


Fig. 72. Vertical acceleration (For'd) and its derivative Runs 1 & 2 27 OCT. 76.



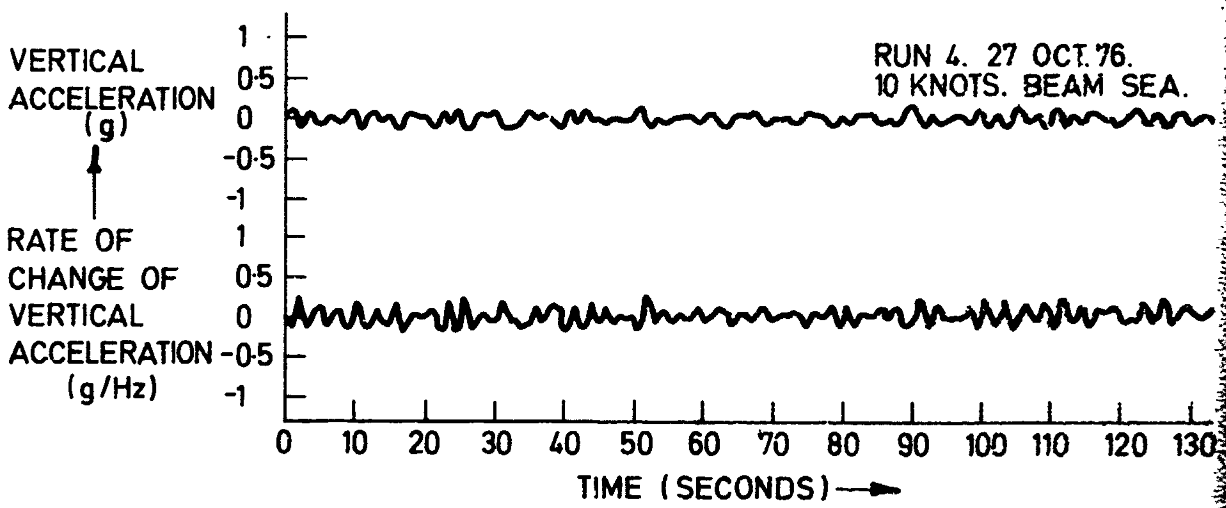
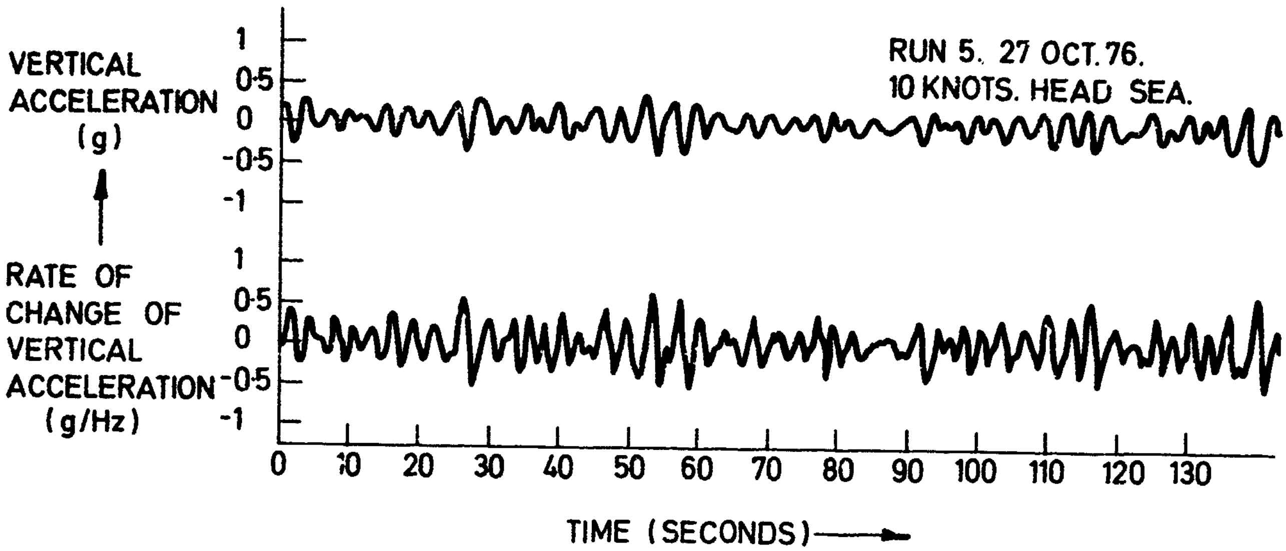


Fig.73. Vertical acceleration (For'd) and its derivative Runs 3 & 4 27 OCT. 76.



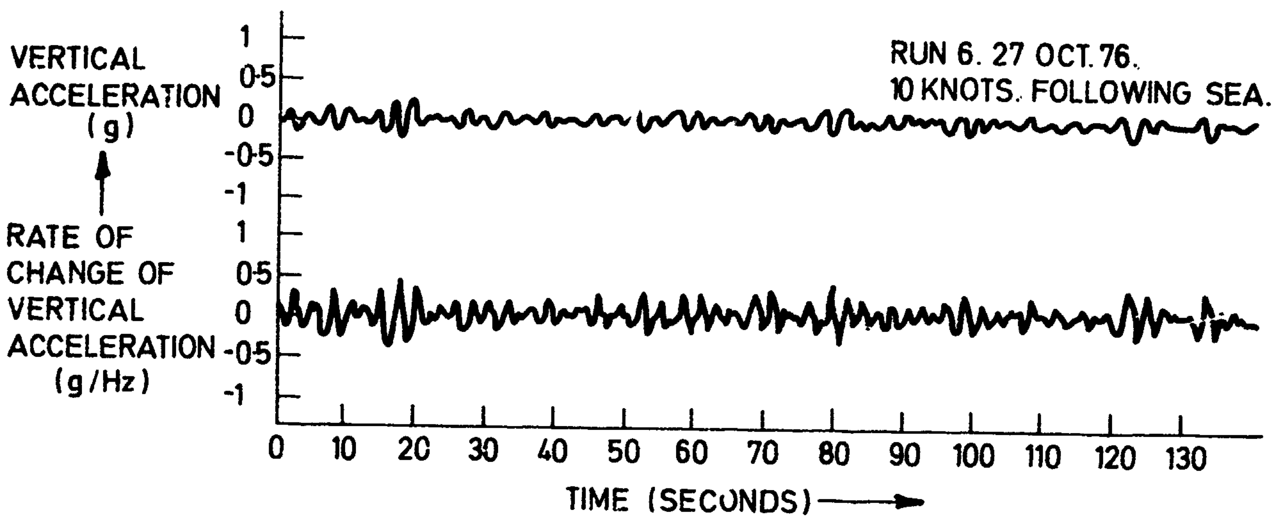


Fig.74. Vertical acceleration (For'd) and its derivative Runs 5 and 6 27 OCT. 76.

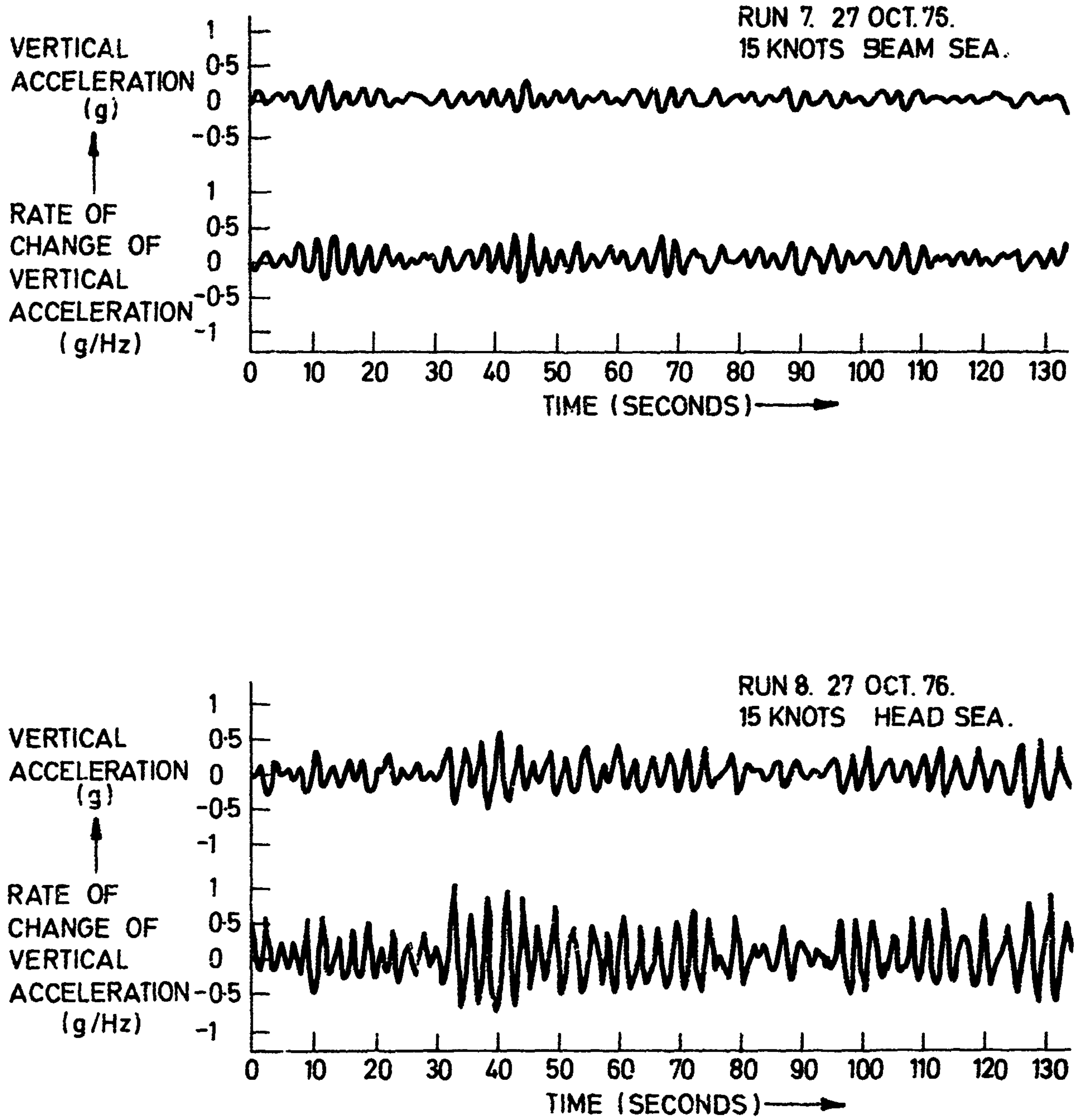


Fig.75. Vertical acceleration (For'd) and its derivative Runs 7&8 27 OCT. 76.

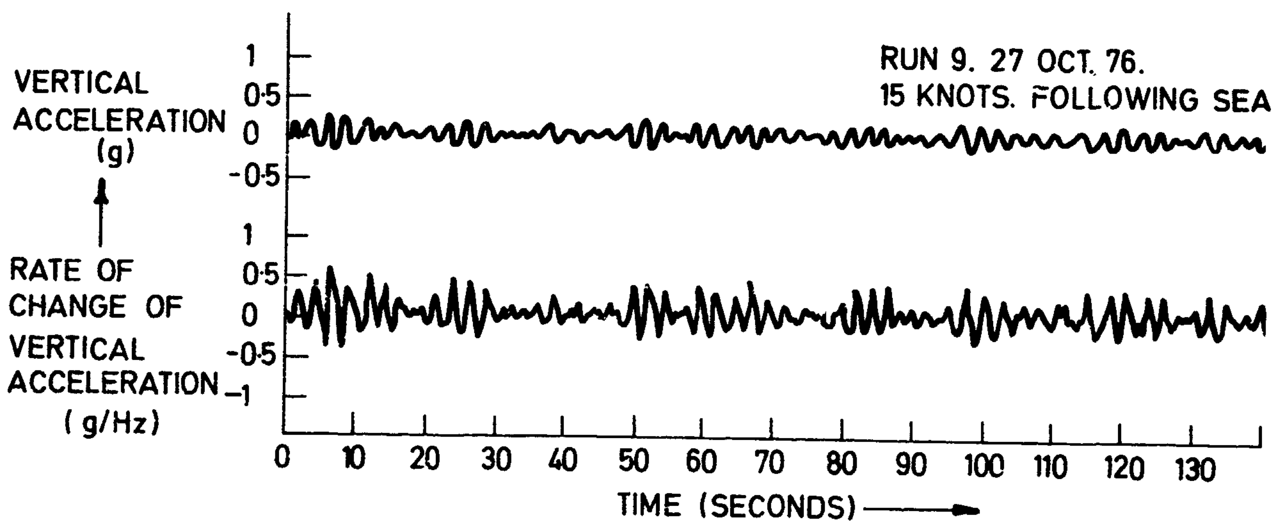


Fig. 76. Vertical acceleration (For'd) and its derivative Run 9 27 OCT. 76.

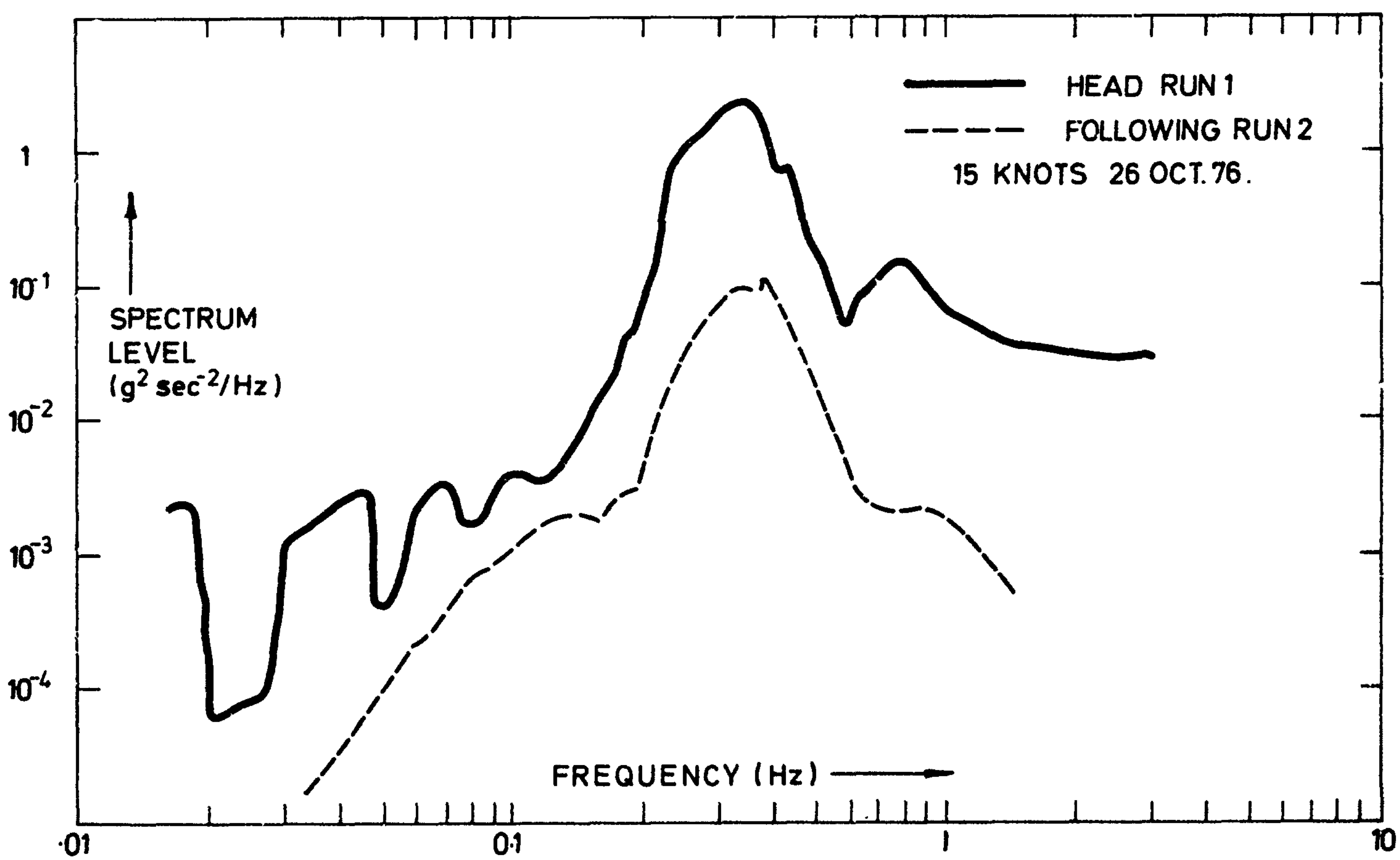


Fig. 77. Spectra of rate of change of vertical acceleration at 15 knots (For'd) Runs 1 & 2 26 Oct. 76.

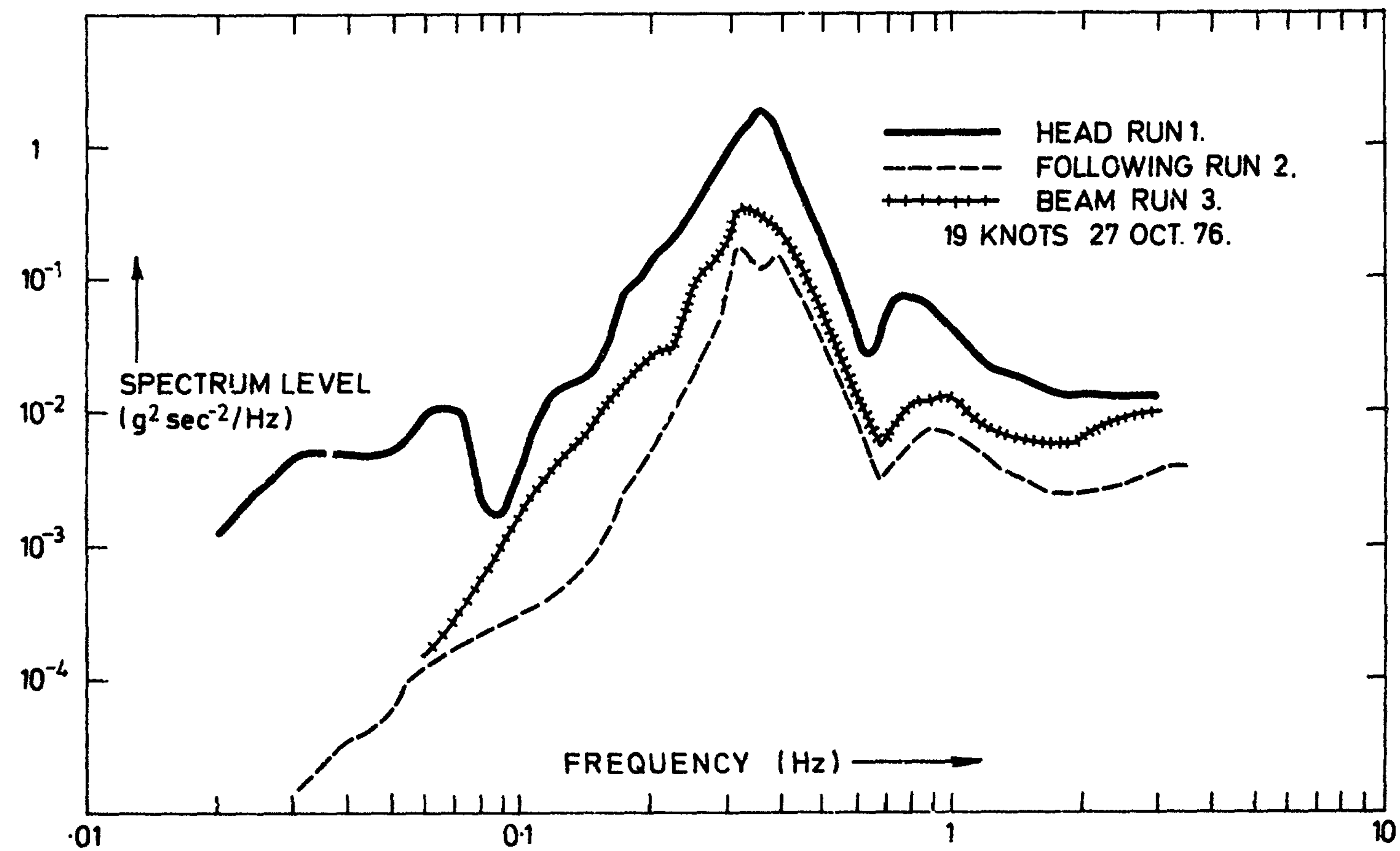


Fig.78. Spectra of rate of change of vertical acceleration at 19 knots (For'd) Runs 1,2,3 27 Oct. 76.

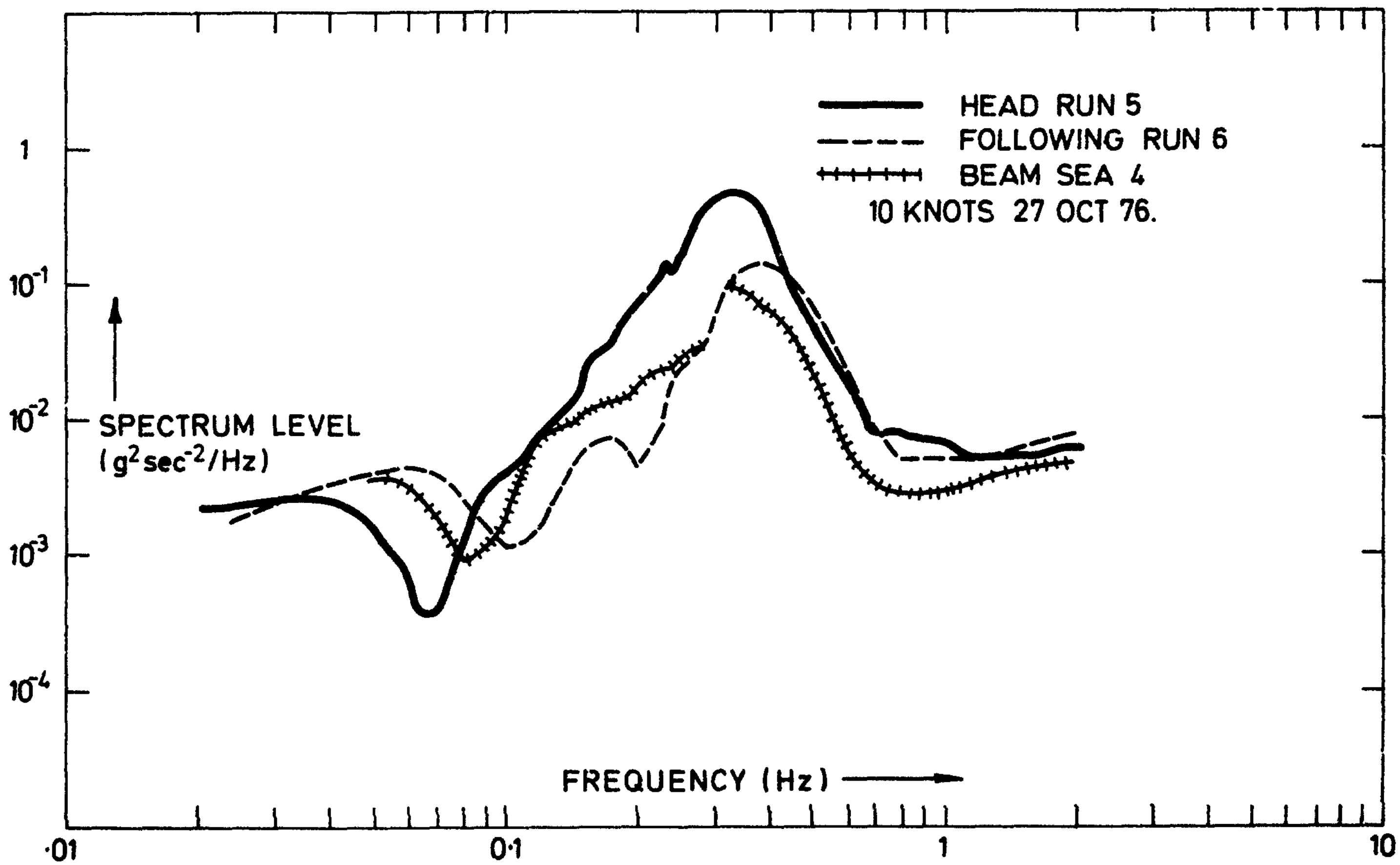


Fig.79. Spectra of rate of change of vertical acceleration at 10 knots (For'd) Runs 4,5,6 27 Oct. 76.

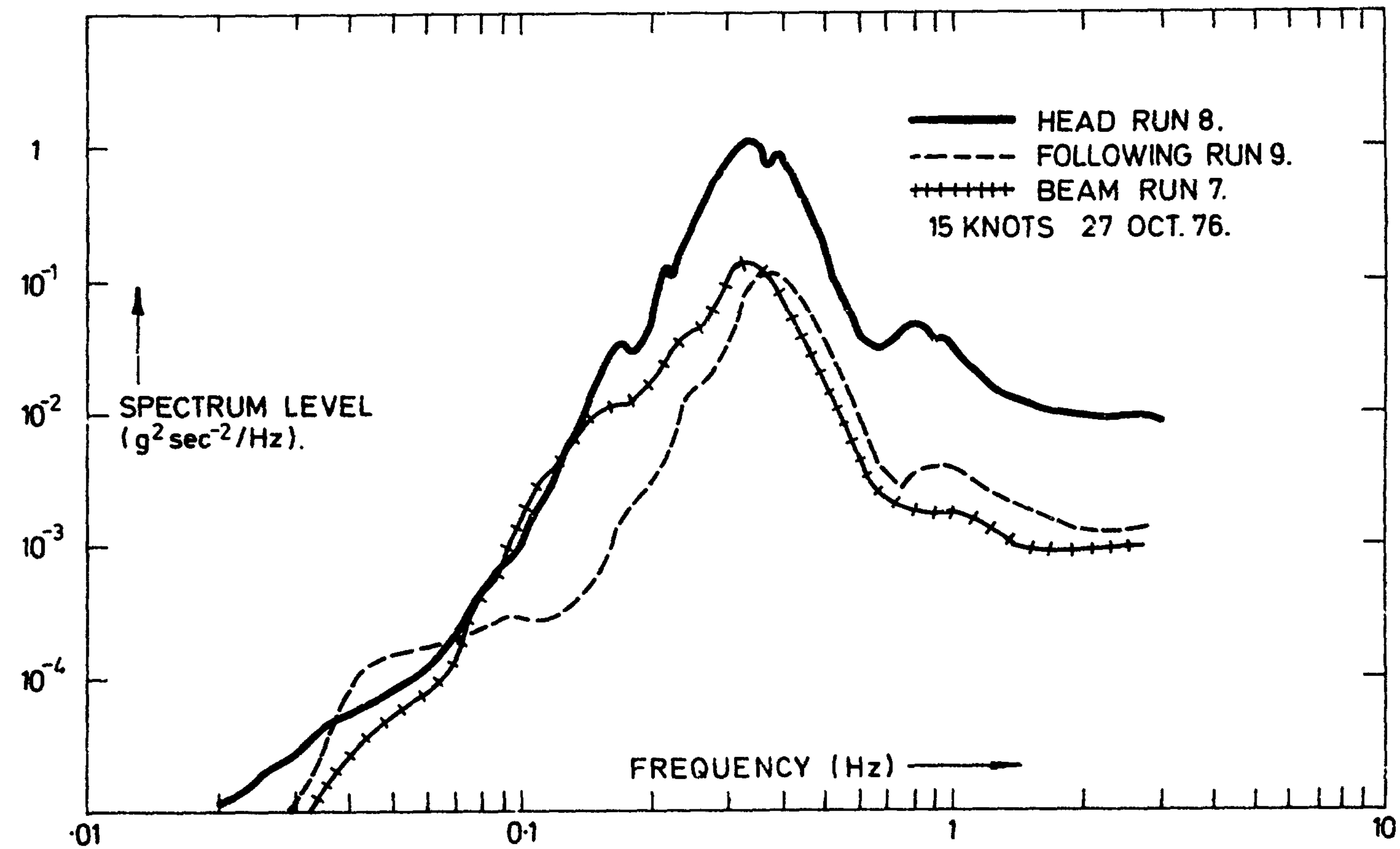


Fig.80. Spectra of rate of change of vertical acceleration at 15 knots (For'd) Runs 7,8,9 27 Oct. 76.

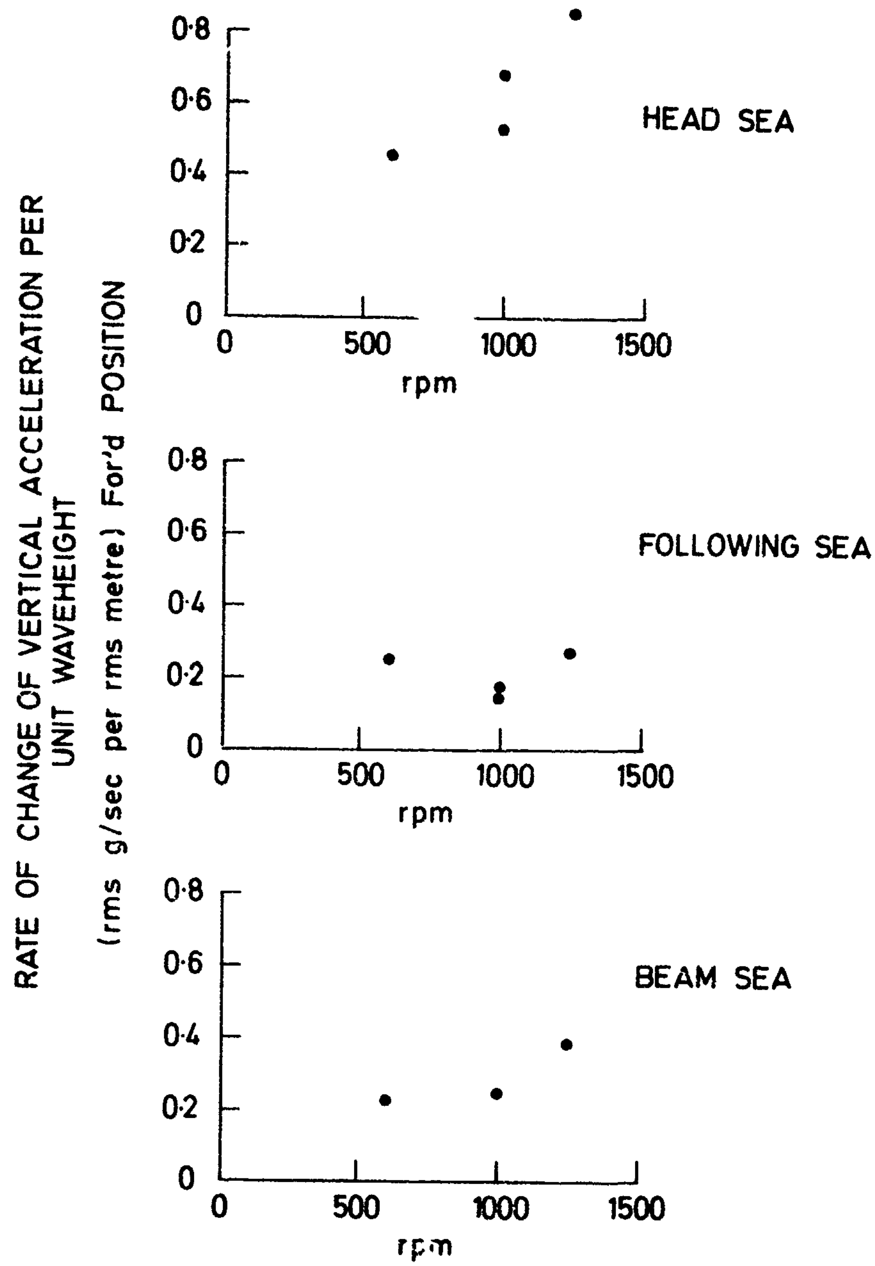


Fig. 81. Rate of change of vertical acceleration per unit waveheight.

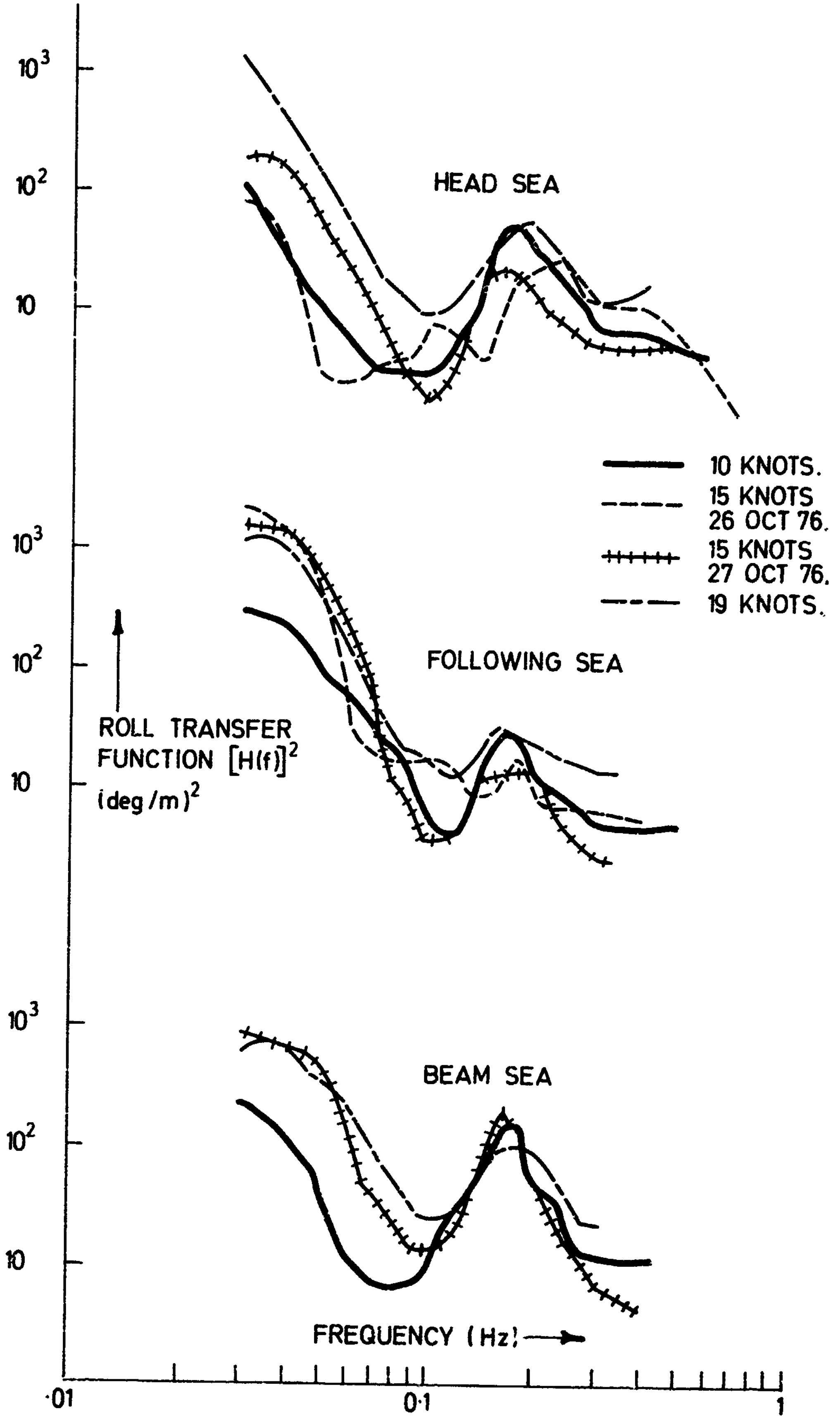


Fig.82. Roll transfer functions.

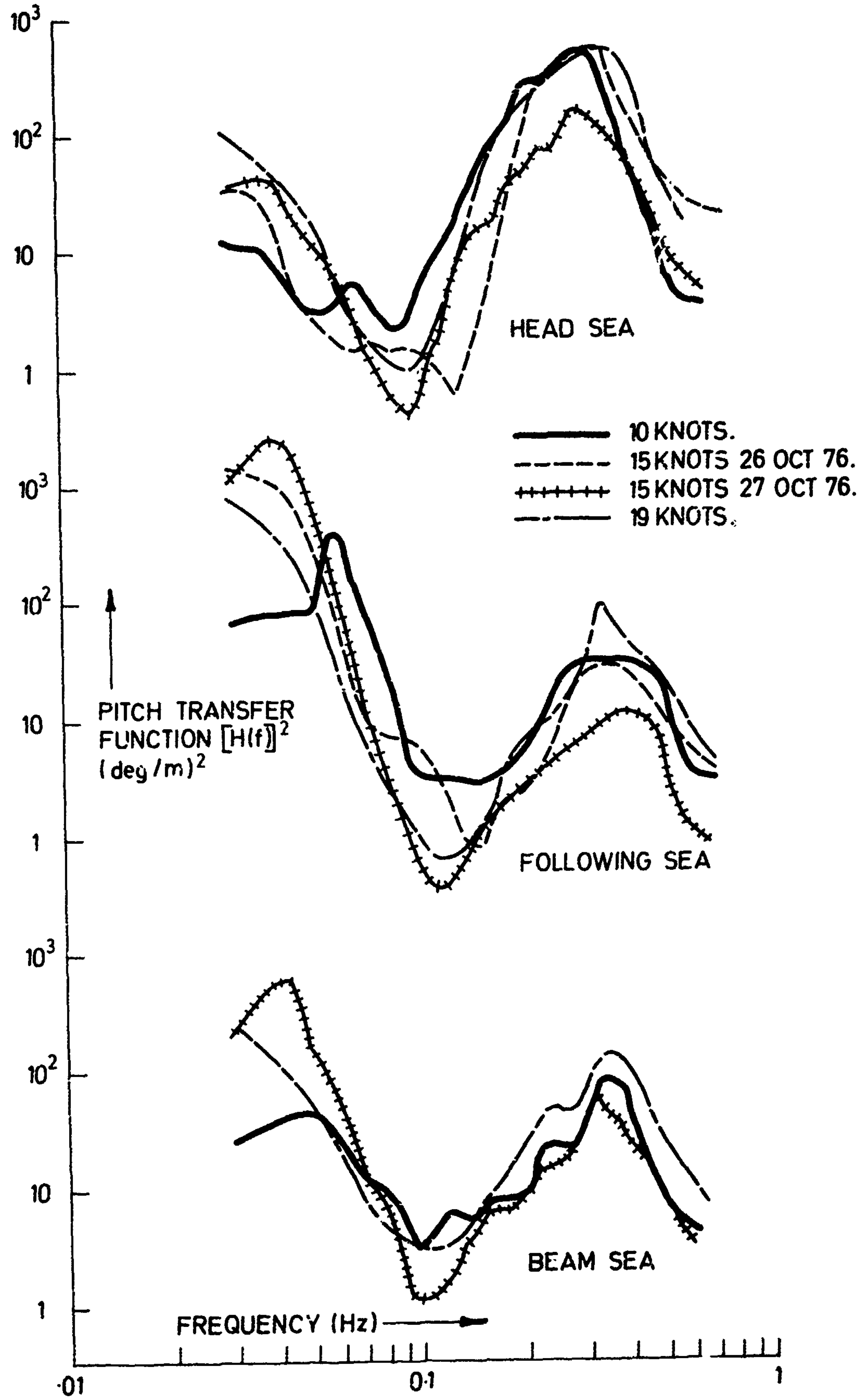


Fig.83. Pitch transfer functions.

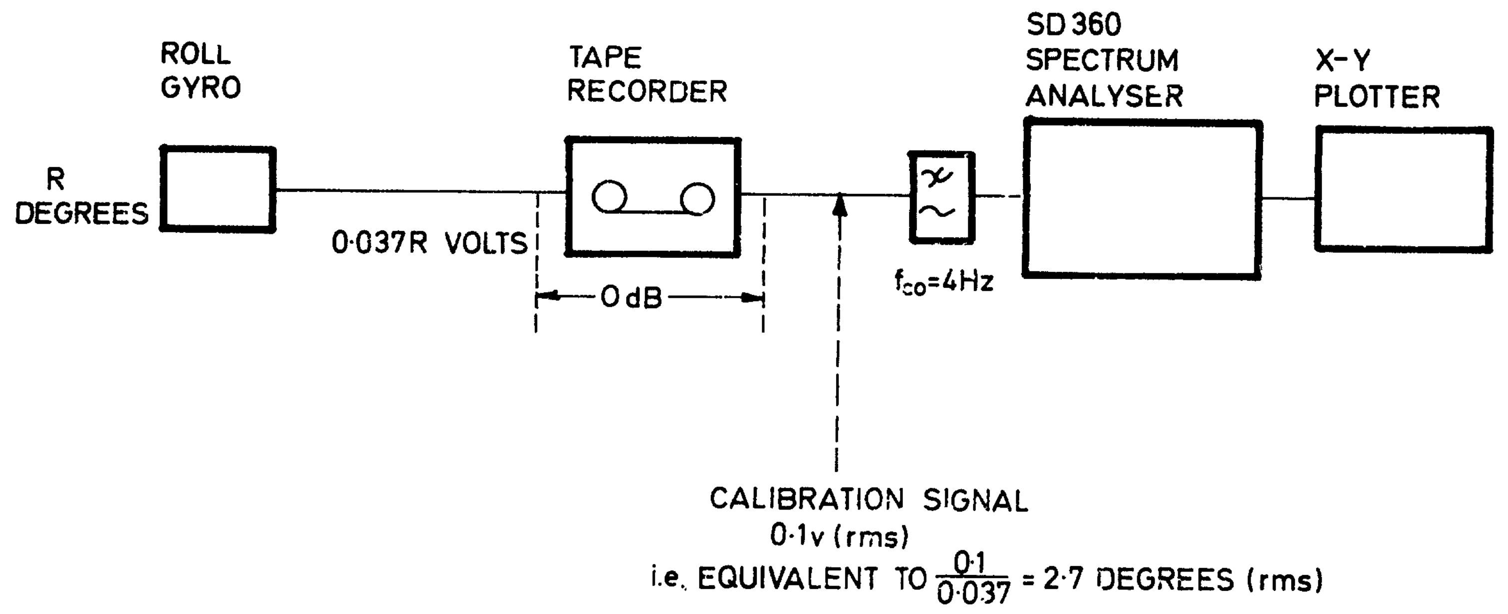


Fig. A1. Instrumentation to measure roll spectra.

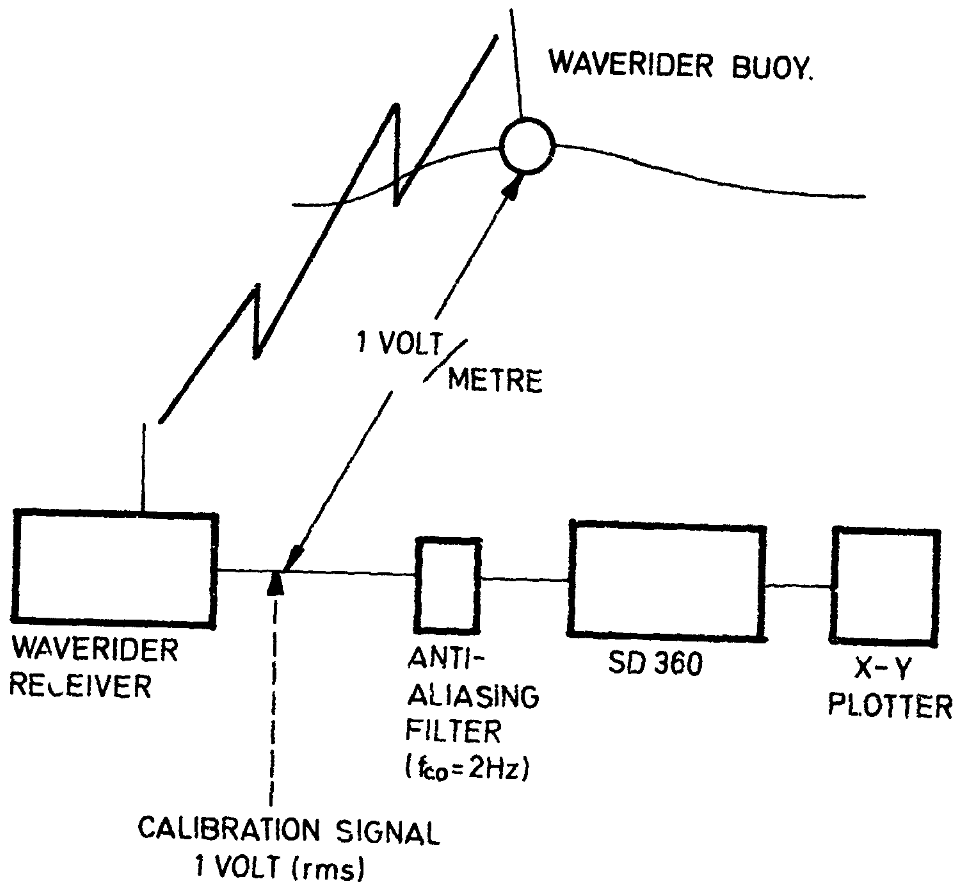


Fig. B1. Instrumentation at HMAS Watson to measure waveheight spectra.

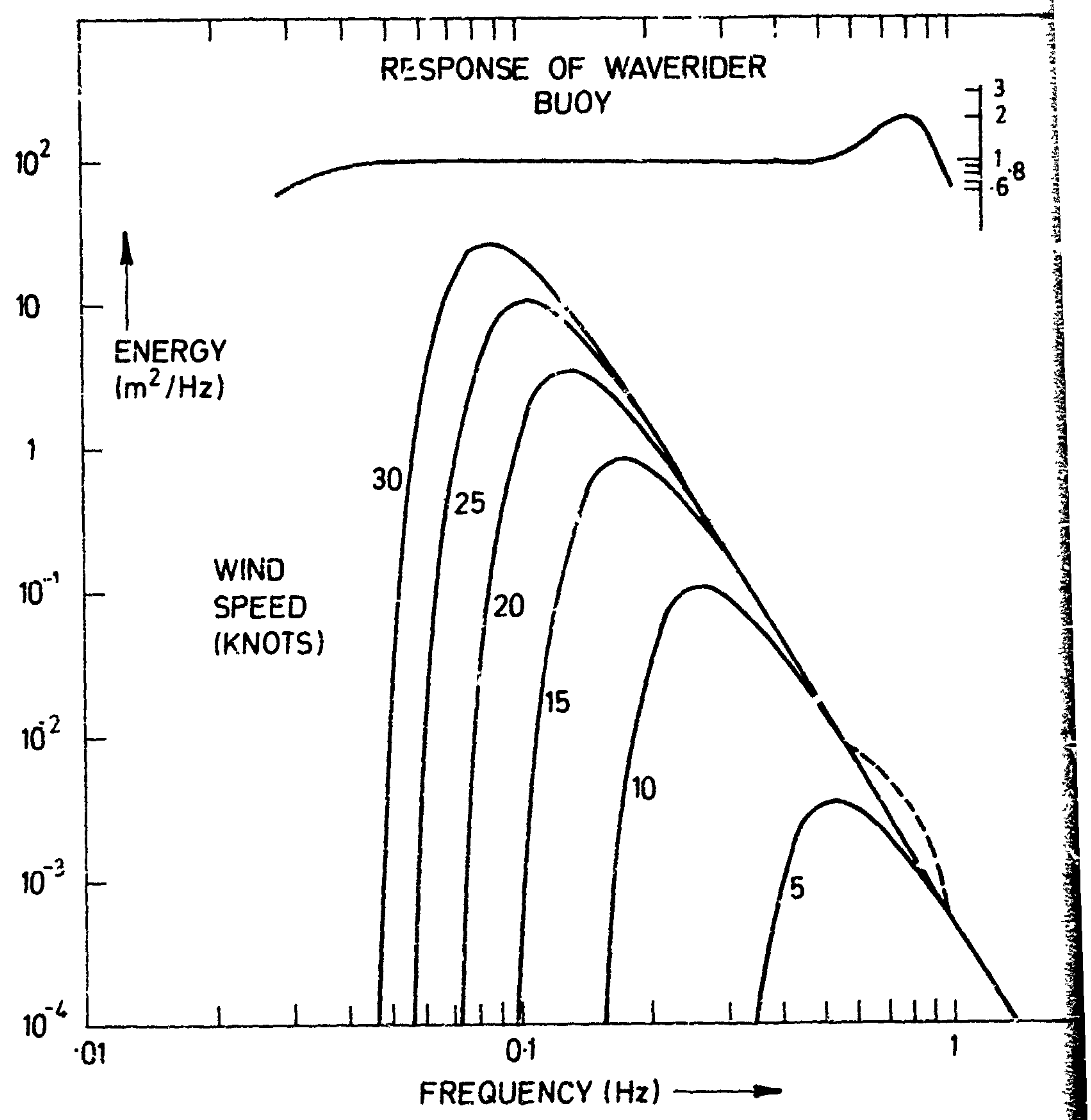


Fig. D1. The Pierson-Moskowitz spectrum for a fully developed sea.